# On the Origins of the Cartouche and Encircling Symbolism in Old Kingdom Pyramids 

## David Ian Lightbody



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Frontispiece (above). Representation of a "cosmic frame" panel inscribed on a travertine/ Egyptian alabaster jar dedicated to Pepi I Meryre, on the occasion of his first heb-sed (Egyptian Museum Berlin, ÄM 7715). The text reads, from right column to left and from top to bottom: "The beautiful pyramid of Pepi is enduring, the living Horus, beloved of the two-lands, nswt-bity, Pepi, son of Re. An offering of never-ending encircling protection, life, and dominion; given life, stability, and dominion, forever. On the occasion of the first heb-sed". This panel demonstrates the close relationship between the pharaoh's names, cults, and architecture. The architectural aspect is manifested by the palace façade serekh and by the name of the pyramid. The symbolism of the piece alludes to the dual eternal and ephemeral nature of the pharaoh, and the vessel would have been used in the pharaoh's cult at the new pyramid complex. It was found in his mortuary temple in South Saqqara (D. Lightbody).

Cover image. Hieroglyphs within a shen-ring: šnnt $m r$ hwt-ntr; stating that "[This] pyramid and temple are encircled". The text is from Pyramid Text PTं 534 §1277, found at the entrance to the pyramid of Pepi I Meryre in South Saqqara (D. Lightbody).

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## Conventions

After the introduction, the text and chapters of this book are arranged in chronological order. The illustrations of the relevant artifacts and monuments are distributed through the text and are numbered according to the dynasty in which the piece or building was created, and then by the order in which the item is discussed in the chapter. For example, (Figure 2-1) refers to item 1 in the chapter covering dynasty 2 . The depictions, therefore, correspond rigorously to the sections covering the dynasty in which the subject matter was created. Towards the end of the book is an additional section illustrating and discussing related artifacts from the Middle and New Kingdoms. These later examples, which were not created during dynasties 1-6 that are the focus of this study, all have 7 as the first digit in the bipartite figure code. That number does not actually refer to dynasty 7 and is only used to maintain chronological order. The table of figures on the previous two pages sets out the titles of the illustrations, the order in which they appear, and the corresponding figure numbers.

The acronyms B.C. and A.D. are used throughout. This is the author's preference, due to the force of habit, and it is retained on the basis that C.E. and B.C.E. seem to lack some degree of clarity, and take up more space on the pages and in tables.

The terms "pharaoh" and "pharaonic" are used extensively in this work to refer to the rulers of Upper and Lower Egypt and their culture, for the whole dynastic era. The Egyptian roots of the English word pharaoh, pr- -3 "great house", were not used to refer to a person until the New Kingdom, and so referring to the ruler as a "pharaoh" or anything as "pharaonic" may not seem appropriate for the Old Kingdom period. The alternative English word "king", however, was never used to refer to any Egyptian ruler during Antiquity. As will be discussed in this current work, the ancient Egyptian system of leadership was intimately tied to the landscape, history, and culture of the lower Nile Valley in a way that did not apply in other regions. The use of a unique term that has its roots in ancient Egypt, and which alludes to the architecture of ancient Egypt as well as the Egyptian ruler, means that the word "pharaoh" is most appropriate for the current context. The word "pharaonic" refers to the unique organizational system in which the "pharaoh" existed.

## Abbreviations

| AIA | Archaeological Institute of America. |
| :--- | :--- |
| ASAE | Annales du Service des Antiquités de l'Égypte. |
| AUC | American University in Cairo. |
| BIFAO | Bulletin de l'Institut Francais d'Archéologie Orientale. |
| BiOr | Bibliotheca Orientalis. |
| BMFA | Bulletin of the Museum of Fine Arts. |
| BSAE | British School of Archaeology in Egypt. |
| CAJ | Cambridge Archaeological Journal. |
| DE | Discussions in Egyptology. |
| EES | Egypt Exploration Society. |
| GM | Göttinger Miszellen. |
| IFAO | l'Institut Francais d'Archéologie Orientale. |
| JAEA | Journal of Ancient Egyptian Architecture. |
| JARCE | Journal of the American Research Center in Egypt. |
| JEA | Journal of Egyptian Archaeology. |
| JNES | Journal of Near Eastern Studies. |
| JSSEA | Journal of the Society for the Study of Egyptian Antiquities. |
| MDAIK | Mitteilungen des Deutschen Archäologischen Instituts, Abteilung Kairo. |
| MDOG | Mitteilungen der Deutschen Orient-Gesellschaft, Berlin/ Leiprig. |
| OMRO | Oudheidkundige mededelingen van het Rijksmuseum van Oudheden in Leiden. |
| PM | Porter, B. and Moss, R., Topographical Bibliography, Oxford, Griffith Institute. |
| ZÄS | Zeitschrift für ägyptische Sprache und Altertumskeunde. |

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It has been the greatest privilege of my intellectual life to work on this fascinating subject. Due to the potential significance of the conclusions drawn, I decided to share the publication as widely as possible by publishing Open Access. I hope that this treatise is an adequate tribute to the ancient monument builders, to the many people who have helped me study them, and to all you readers who have decided to learn more about them. It is my sincere hope that you take the time to carefully evaluate all of the information presented here for yourselves, and that you enjoy the experience as much as I have.

This book is dedicated to Carl and Carlanne Herzog's Christian book group in Vermont, which has been particularly supportive of my ongoing research and publication efforts.

David Ian Lightbody, Ph.D., M.Phil, BEng (Hons), PGCE. Vermont, 2019.

## Foreword

The subject of this publication is encircling symbolism in pharaonic monumental tomb architecture. The study focuses on the Early Dynastic Period and the Old Kingdom; from the first dynasty through the sixth dynasty of ancient Egyptian history. During that time, encircling symbolism became most influential and was developed most significantly. The cartouche also became the principal symbol of the pharaoh for the first time. This work demonstrates how the development of the cartouche was closely related to the monumental encircling symbolism incorporated into the architectural designs of the Old Kingdom pyramids.

This publication builds on a long-term research project that I began in 2004, and which in turn built on research and knowledge assembled by other scholars who worked on the subject during the $19^{\text {th }}$ and $20^{\text {th }}$ centuries. Far from being resolved or settled, the subject matter was still poorly understood when I first encountered it in 2004. In 2008, I published a monograph ${ }^{1}$ setting out my preliminary thoughts on the issue, which are more fully elucidated here with corrections and additions where necessary.

The first objective of the 2008 report was to compile and publish the basic archaeological evidence that was available at that time, predominantly in the form of linear measurements derived from standing building surveys. The second objective of the publication was to demonstrate how the data set assembled there supports the proposal that the ancient Egyptians utilized special dimensions and proportions ${ }^{2}$ to build their pharaonic monuments. As described below, some of the principal dimensions and proportions of the pharaonic tombs were based on the geometric properties of circles. The third objective of the 2008 study was to investigate the symbolic context of that architectonic tradition, in order to understand what it meant in the ancient Egyptian mind. All three of those objectives were met, but with respect to the third objective, the analysis was superficial, only hinting at what subsequently proved to be a deeply important pharaonic tradition.

As will be discussed below, encircling symbolism was expressed simultaneously through multiple aspects of ancient Egyptian culture. It manifested through the ancient Egyptian language, through their scripts, in the decoration of their jewelry and fine vessels, in their architectural decor, as well as in the principal dimensions of their most significant pharaonic monuments.

It was important to revisit this subject in a new and dedicated publication because it elucidates aspects of the early development of the abstract sciences and applied engineering, including architecture and math, in ancient Egypt. ${ }^{3}$ Despite its importance, the subject area has remained

[^0]confused, ${ }^{4}$ and at times even contentious, ${ }^{5}$ and the technical discussions lacked clarity. ${ }^{6}$ This updated publication presents and elaborates the hypothesis put forward by Petrie and others. It is hoped that by compiling and developing the latest available information into an organized and useful research report that the subject can be more readily appreciated by interested readers. ${ }^{7}$

4 In the late $19^{\text {th }}$ century, the issue of circular symbolism in the Great Pyramid was taken up by the Astronomer Royal for Scotland, Charles Piazzi Smyth. His theories on the matter were distorted by his religious and nationalist beliefs and it was not until Petrie's survey became available in 1883 that most of Smyth's ideas were effectively debunked. Smyth's publications continued to confuse the matter and discredited the issue to some extent in the public mind. Petrie did provide a clear explanation of the architectural phenomenon as he saw it, and the issue was rehabilitated to some extent by the mid- $20^{\text {th }}$ century. I.E.S. Edwards was one of those Egyptologists who broadly accepted Petrie's conclusions on the matter.
5 At the end of the $20^{\text {th }}$ century, the issue of circular symbolism in the architecture of the pyramids became one of the core issues discussed during the debates surrounding the contribution of Africa to the development of the sciences. A series of contentious articles were published as part of the "Black Athena Debate", focussing on the hypothesis published by Martin Bernal, grandson of Egyptologist Alan Gardiner. See M. Bernal, Black Athena: The Afroasiatic Roots of Classical Civilization (London: Free Association Books, 1987). The issue of circular symbolism was drawn into that debate, but the discussions were inconclusive and remained confused as they primarily referenced philological works rather than the architectural evidence from the monuments themselves.
6 One of the most widely read and well developed discussions of the issue of circular symbolism in the Great Pyramid's architecture in recent years was the publication by C. Rossi, Architecture and Mathematics in Ancient Egypt (Cambridge: University Press, 2003). I addressed the arguments made there in my 2008 publication, and the Addendum of the current publication addresses the mathematical evidence in more detail. The conclusions of my analysis indicate that the written evidence from mathematical papyri and the architectural evidence from field surveys are in fact complementary, rather than contradictory, and do support the conclusions reached by Petrie that the circular symbolism was intentionally incorporated into the designs of the monuments.
7 Over the course of the research project, I published shorter interim presentations of aspects of the research, including D. Lightbody, "The Encircling Motifs of Old Kingdom Avian Themed Pharaonic Vases", GM 249 (2016); "Biography of a Great Pyramid Casing Stone", JAEA 1 (2016); "The Encircling Protection of Horus", in Proceedings of the Twelfth Annual Symposium Current Researches in Egyptology, 2011, University of Durham, eds. H. Abd El Gawad, et al. (Oxford: Oxbow, 2012).

## Introduction

This introduction sets out the theoretical basis and background to the study, before the archaeological information is presented in chronological order in the subsequent sections.

To be rigorous, research must take place within an established academic context and a rational intellectual structure. These provide the organizational and theoretical foundations of all advanced study. In order to contextualize this current study for the reader then, the organizational and theoretical background against which it was carried out must be defined and summarized. The first point of note is that the research was inherently interdisciplinary in nature. In order to understand the core issue, every available class of evidence from the Old Kingdom had to be examined. All of the available publications relating to the period and subject in question, including scholarship from several different sub-fields of study, were consulted. The different issues that arose in those sub-fields are described below.

The most basic approach taken towards understanding the Old Kingdom is to address it as history. Traditionally, the study of history began by following the "great man theory". ${ }^{8}$ In Egyptology, the approach translates into interpreting ancient Egyptian history as a sequence of events shaped by a sequence of influential pharaohs and their dynasties. The current study does follow a relatively traditional diachronic approach, and the role of pharaohs in the ancient Egyptian culture was clearly significant, but like many other historians I have questioned the basis of this approach. ${ }^{9}$ The genesis of influential men or women depends on factors specific to the cultures that produced them, and on the specific context into which the individuals were born. Before "great men" or women could remake society, society had first to make them. Later social historians viewed historical changes with a much longer perspective, ${ }^{10}$ and some considered that cultures undergo specific periods of profound change as a result of multiple influences that intersect during specific periods, rather than depending on the transformative actions of any one individual.

The architectural iconography studied here was developed by the pharaonic culture, and while it remains difficult to interpret it with respect to specific individuals, ${ }^{11}$ it is possible to relate important changes that appear in artworks to specific historical events and cultural changes taking place at the time they were made. In this study of pharaonic Egypt, it appears that the most significant changes were initiated by the Old Kingdom court, by its artisans as well as the members of the administration, and perhaps at times by the pharaoh himself. The start of the third dynasty was a phase of particularly intense cultural change. In the pages that follow, I propose that many novel aspects of the third dynasty's monumental architecture and iconography were deliberately and meaningfully developed as the result of a newly emerging political reality. We can better understand the political history of the era by understanding the changes visible in the artistic canon, but the analysis must encompass the whole of the pharaonic culture in question, rather than the lives of individual pharaohs or artisans.

[^1]Symbolism in ancient Egyptian architecture is most conventionally considered to fall within the realm of art ${ }^{12}$ and art-history. John Baines considered that symbolism in Old Kingdom material culture is most meaningfully interpreted using concepts from the theoretical study of art and architecture. He stated that the funerary monuments in Old Kingdom Egypt were comprehensively planned as works of art, ${ }^{13}$ and that architecture was the core genre of artistic expression for the emerging state. ${ }^{14}$ This was particularly the case in the dynasties before writing became the predominant mode of communication. ${ }^{15}$ Richard Wilkinson, on the other hand, considers that little of Egyptian artwork can be considered as "art for art's sake", and that most artworks were conceived within a matrix of symbolism and magic. For him, ancient Egyptian artworks cannot be fully comprehended without knowledge of the underlying concepts intrinsic in their composition. ${ }^{16}$ Research must, therefore, include approaches that can deal with concepts such as symbolism and magic in architecture. Robert Ritner, who studied Egyptian magic and religion in depth, stated that ritual activities were not felt to be supernatural, but to be quintessential parts of nature, and were thought to be used daily by the gods to maintain, not violate, the natural order. ${ }^{17} \mathrm{He}$ also showed that encircling magic was relatively common in ancient Egypt. It was applied in contexts as simple as casting water purification spells for liquid-filled vessels, or as elaborate as ritual circumambulations performed during major festivals. By understanding these magical and ritual meanings, and by applying this understanding to the architectural and artistic contexts, the meanings being expressed by architectural design motifs can be revealed.

The culture-history approach considers the domain of artistic representation and the struggle over meaning as the most fruitful avenue for achieving historical understanding of a culture. In this respect it shares some common ground with anthropological approaches that consider cultures to consist of structures of meaning. According to Baines, a small number of schematic elements usually characterize a civilization. ${ }^{18}$ One such "schema" used in ancient Egypt was the group of inter-related and essential symbols that became central characteristics of the pharaonic culture, including the 「 $n h$ ankh, $w 3 s$ was scepter, $\underline{d} d$ djed column, srh serekh, and $h r$ pharaonic falcon, which are referred to here as a "symbolic repertoire". ${ }^{19}$ As the premier medium of artistic expression during the Old Kingdom, architectural designs were integral to that cultural schema. Baines also considers that ground plans of monuments such as temples could be representational schemas. ${ }^{20}$ That concept is adopted here and put forward to help explain the inclusion of such monumental motifs in Old Kingdom pharaonic architecture.

More traditional archaeological approaches also provided hard data sets for the current study in the form of linear measures from standing building surveys. Empirical information was also compiled regarding the materials used in construction and to fabricate smaller artifacts, and radiocarbon dating information was used to construct a chronology of key events.

[^2]More progressive archaeological methods also proved useful. Archaeology now tries to contextualize people and their cultures within specific habitats and cultural landscapes. This approach is particularly useful for studying sites predating the fourth dynasty, from the protoliterate era when the performative aspects of culture predominated. Ritual was the principal means of elite communication at that time, and both architecture and artwork reflected the ritual activity underpinning the early pharaonic system. ${ }^{21}$ It can be anticipated that new monuments constructed during that era, such as pharaonic tombs, were designed as new places to establish and maintain traditional cults, and that they were designed with the rituals in mind. The pharaoh's funerary monuments incorporated the symbolism of the mortuary rituals at a profound level. An understanding of the wider ritual context in which the art and architecture were constructed can, therefore, provide a way to access the meanings being expressed in those domains.

The monumental architecture of Old Kingdom Egypt incorporated reliefs and statues. These were carefully placed within temples and carefully designed to be experienced with respect to the surrounding architecture. The monuments as a whole were located with respect to the local cultic topography and with respect to the wider Egyptian landscape and the heavens above. The architecture even reflected aspects of the natural environments that surrounded the temples. As will be seen, even the local zoological and botanical contexts were influential, and structured symbolism relating to these can be identified in the designs of Old Kingdom pharaonic architecture and iconography.

A final research approach used in this study was philology. Towards the end of the Old Kingdom, hieroglyphic writing began to play a more prominent role in conveying ideology previously carried by monumental architecture, iconography, and ritual. The analysis of ancient texts falls within the field of philology, and although philologists tend not to interpret texts with respect to architecture, their studies provided important information about architectural motifs and rituals, as well as about the mathematical procedures ${ }^{22}$ used by the scribes and architects when designing the monuments.

The main symbol examined in this treatise is the shen-ring, including in its elongated form known as the cartouche. The textual meaning carried by the shen symbol is often reduced to "eternity", "the whole world encircled", or "everything encircled by the sun and the king's dominion over it". ${ }^{23}$ Several in-depth philological analyses of the symbol have appeared in recent years, ${ }^{24}$ and other definitions are available. ${ }^{25}$ In later dynasties, associated meanings became more varied and

[^3]the symbol appeared in many different contexts, but it seems that its Old Kingdom sense was originally derived from the word "to encircle". ${ }^{26}$ The symbol was closely linked to architectural contexts through the ritual concept of "unending encircling protection", most often surrounding royal tombs and enclosures. The associated shen word family supports this interpretation, ${ }^{27}$ as do the architectonic case studies outlined in the chapters that follow.

Anthes discussed meanings associated with the word shen when used as a verb within Old Kingdom texts. They ranged from "enchant", to "captivate", "encircle", "hold", "embrace", "capture", "enclose", and "spellbind". For the Pyramid Texts of the Old Kingdom, he assigned a special meaning of "encircle", "captivate", "hold", or "bind". Importantly, Anthes also concluded that, as a noun, the term referred to a physical enclosure or circle. ${ }^{28}$

Many written signs in the ancient Egyptian hieroglyphic script represented or resembled real objects, ${ }^{29}$ while others were abstract shapes that represented the sounds of syllables or groups of syllables. Other glyphs known as determinatives signified words associated with specific groups of ideas. A fourth, intermediate, type of representation existed in iconography where a symbol did not realistically depict any real object, but referenced a specific abstract concept or ritual. These can be referred to as emblems. ${ }^{30}$ It is within this class of symbol that the shen-ring most realistically belonged when it was used in iconographic arrangements. Rituals themselves are ephemeral, and because of their special status, access to their meanings was often restricted. They were not usually depicted explicitly, but in a way that reflects their symbolic value. ${ }^{31}$ Given its prominence in scenes representing rituals in Old Kingdom pharaonic monuments, the shen-ring appears to have held a special status within those rituals.

The approaches outlined above led to conclusions that do not necessarily conform to traditional understandings of the symbols and the monuments being studied. Nevertheless, the interdisciplinary and contextualized approach taken here has perhaps proved to be more fruitful than the rather dry culture-historical approach, ${ }^{32}$ or less rigorous art-historical approach, when those paths are taken in isolation. Older studies tended to be overly focused on tracking the use of motifs through time, back to "original" essential meanings that were assumed to be unchanging. They have not, however, provided any explanation for some of the most fundamental questions in Egyptian history, such as

26 R.J. Leprohon, The Great Name in Ancient Egyptian Royal Titulary, vol. 22, Writings from the Ancient World (Atlanta: Society of Biblical Literature, 2013), 8.
27 Definitions for $\check{s} n w$ include perimeter, enclosure wall, enclosure. See A. Erman and H. Grapow, Wörterbuch Der Ägyptischen Sprache, Vol. IV (Berlin: Akademie Verlag, 1971), 488, 489, 491; R.O. Faulkner, A Concise Dictionary of Middle Egyptian (Oxford: Griffith Institute, 2002), 267-268. Further research should be carried out to better understand the meanings associated with the word šn in Middle Kingdom and New Kingdom sacred texts, i.e. the Coffin Texts and Book of the Dead, respectively, to understand if and how the nuances changed over time. That work lies outside the scope of the current study and beyond the capabilities of the current author, and should be undertaken by specialists in the ancient Egyptian language.
28 R. Anthes, "Das Verbum Sni „Umschließen, Bannen" in Den Pyramidentexten", ZÄS 86 (1961), 89.
29 Bastioned enclosures and palace-façade walls were often represented in early hieroglyphs, including on ceremonial palettes and seals. Some of these were ovoid and so visually resemble the cartouche to some extent. The different encircling symbols representing $d t$ estates, forts, towns, and inbw walls have been discussed at some length. See J. Monnet-Saleh, "Forteresses, Ou Villes-Protégées Thinites?", BIFAO 67 (1967); Spieser, Les Noms Du Pharaon: Comme Etres Autonomes Au Nouvel Empire, 22.
30 Baines, "Communication and Display: The Integration of Early Egyptian Art and Writing", 474; idem, "Temples as Symbols, Guarantors and Participants in Egyptian Civilization", 218. Renfrew describes a similar concept associated with the emergence of material symbols in non-literate sedentary societies. He refers to these as "constitutive symbols", which do not necessarily represent something else, but are themselves active. They allow and facilitate the emergence of institutional facts, such as structured kinship relations or power hierarchies. The emblems that developed in proto-literate pharaonic Egypt seem to fit within this theoretical framework. See Renfrew, 130.
31 Baines, "Communication and Display: The Integration of Early Egyptian Art and Writing", 476; idem, "Temples as Symbols, Guarantors and Participants in Egyptian Civilization", 223.
32 B. Trigger, A History of Archaeological Thought (12th Printing) (Cambridge: Cambridge University Press, 2004), 200.
why the cartouche quite suddenly became the principal symbol of the pharaoh in the third dynasty, or why the serekh was no longer used to contain the pharaoh's name after he had died. Answers to those questions are proposed below.

In conclusion, this study takes an inter-disciplinary, contextualized approach that addresses the encircling symbolism from several different angles. This holistic approach draws out meaning from the symbolic arrangements and it is informed by theory from several different fields of study. Worked together, concepts from art-history, anthropology, archaeology, and philology can complement each other and reveal underlying meanings that existed in the ancient Egyptian mind.

The following chapters present the results of the study, set out in chronological and dynastic order.

## Architectural expressions of encircling symbolism

This section includes a chronologically ordered examination of encircling symbolism as it related to the pharaonic tomb monuments and iconographies of the first through sixth dynasties of ancient Egypt. The relationship between the architectonic designs and the encircling iconography is described and interpreted systematically for those periods. The supporting historical narrative tells the story of the evolution of the encircling motif through time and geographical space.

## Early Dynastic Period (Dynasties One and Two)

Many of the earliest written symbols that survive from the formative period of Egyptian pharaonic history indicate that the ancient Egyptian script was developed by members of the proto-dynastic ruling group and then the early-dynastic court. Wilkinson considers it fair to say that the bulk of surviving early inscriptional evidence represents rituals and ceremonial activities performed by the pharaoh. ${ }^{33}$ At that time, the monumental tomb architecture had not yet developed into the sophisticated forms found during the Old Kingdom, but the schema of ritual symbols was beginning to develop into a formal structure. The hieroglyphs from that early period show how the concept of encircling magic was already merging with pharaonic rituals and symbols.

A limited number of artifacts have been recovered from the Early Dynastic Period that show the origins of the shen-ring in iconography, long before it was adopted as the cartouche or incorporated into heb-sed scenes. A simple heb-sed scene is known from the first dynasty pharaoh Den's reign (Figure 1-1), but it does not incorporate shen-rings as was typical after the 3rd dynasty. ${ }^{34}$ The first known shen is depicted on an ivory tag ${ }^{35}$ discovered in pharaonic Tomb $T$ at Umm el-Qa'ab near Abydos that also belonged to the pharaoh Den (Figure 1-2). ${ }^{36}$ The shen is shown alongside pharaonic signs including Den's serekh, with the typical palace-façade motif surmounted by Horus. The serekh served to enclose and protect the pharaoh's Horus name. ${ }^{37}$ This is flanked by a coiled uraeus snake and the hieroglyph for gold, $n b w$, possibly a reference to the pharaoh's Golden Horus name.

A more comprehensible early group of signs including shen-rings was found inscribed on a set of stone bowls and fragments of bowls from Hierakonpolis, dating to the second dynasty (Figure


Fig. 1.1. Den's heb-sed scene. Ebony year label from his tomb in Abydos, with his serekh on the left (D. Lightbody).

33 R.H. Wilkinson, Symbol and Magic in Egyptian Art, 149.
34 British Museum EA32650 is an ebony oil jar label, restored from two fragments, with a hole for attachment at the top righthand corner. Four registers of inscriptions are bordered on the right by a large 'rnpt' hieroglyph, indicating that the text records events of a particular year. The top register depicts the heb-sed festival, showing Den wearing the double crown and running as part of the ritual, and also seated on a throne in a booth.
35 BM EA35552.
36 W.M.F. Petrie, The Royal Tombs of the Earliest Dynasties. Part II (London: Egypt Exploration Fund, 1901), pl. VII; T.A.H. Wilkinson, Early Dynastic Egypt (London: Routledge, 2001), 207.
37 At Abydos, near the earliest known burial ground for the pharaohs of all Egypt, large mud brick funerary enclosures were built that have been described by the excavators as 'the mysterious enclosures of Abydos'. They proposed that the palace-façade decorated niched walls, up to 11 m in height, were the focus for some of the rituals. The wall around the largest of the surviving structures has been attributed to Khasekhemwy, the last pharaoh of the Early Dynastic Period to be buried at Abydos. It has a double-layered wall, which creates what looks like a processional route around the entire monument. Later sources refer to ritual or ceremonial circumambulations by the king around city walls or temple enclosures. See D. O'Connor, Abydos: Egypt's First Pharaohs and the Cult of Osiris (London: Thames and Hudson, 2009), 179.

2-1). A calcite jar with the name of the pharaoh Khasekhemwy found in the dynastic-era temple ${ }^{38}$ carried an iconographic group that showed the vulture goddess Nekhbet standing before the pharaoh's name in a serekh. The vulture grasps the sm3-t3.wy symbol in one claw, symbolizing the


Fig. 1-2. Den's shen. The first known depiction of a shenring, from his tomb in Abydos, with his serekh on the left (W.M.F. Petrie)


Fig. 2-1. Iconographic arrangement including a shen-ring from the reign of Khasekhemwy (D. Lightbody).
unification of the two lands into one. She crushes a symbol thought to represent the abbreviated word for rebels ${ }^{39}$ within a shen-ring, under the other clawed foot. In that early context, the shen probably expressed the encirclement and captivity of the rebels, reflecting the meanings for the word shen derived from Old Kingdom texts by Anthes. ${ }^{40}$ The arrangement perhaps also indicated that the rebels came from within the pharaoh's court, as they were encircled with what may by then have been a royal symbol.

Whatever the precise meaning being expressed in this scene, it differs to the meanings expressed in the later Old Kingdom reliefs, where the shen implied encircling eternal protection, rather than oppression and entrapment. The same scene was repeated on several second dynasty jars including one made of red granite from the same site at Hierakonpolis. ${ }^{41}$ Another fragment was also found at Hierakonpolis. ${ }^{42}$ Notably, the same arrangement was also found on a jar at Saqqara, close to the Step Pyramid complex, in the geographical area where the shen symbol began to take on a more prominent role during the third dynasty. ${ }^{43}$

This evidence from the Early Dynastic Period demonstrates that the shen was known in Hierakonpolis, the place where the dynastic line of the Horus kings most likely originated, as well as at Abydos, in the cemetery where the rulers of all Egypt were first buried. ${ }^{44}$ Subsequently, it was present at Saqqara, in the Memphite necropolis that became the Old Kingdom's principal pharaonic burial ground.

[^4]
## Third Dynasty


#### Abstract

There is evidence that the pharaohs began to move elements of their tombs and associated afterlife monuments from Abydos north to Saqqara during the second dynasty. By the start of the third dynasty, the pharaoh Djoser had moved all components of his mortuary architecture to the Memphite necropolis. The reason why the royal burials moved north at that time remains unclear. Mastabas of royal officials had been built at Saqqara from the time of the pharaoh Aha in the first dynasty. They were clearly supporters of the pharaoh because the walls of their tombs were adorned with the palace façade motif, a symbol of the pharaoh if only because of the widespread use of royal serekhs throughout Egypt. Over time, the accumulation of mastabas in the Memphite necropolis ${ }^{45}$ demonstrated the growing power of the administration, and while there are no indications of high officials acting against the interests of the pharaoh, this accumulation of grand monuments near the capital city may have been one factor that encouraged Djoser to utilize the stony plateau at Saqqara to build a much larger structure; one that would elevate the status of his tomb above the monuments of the state's officials. It is proposed here that he deliberately instituted a new type of architecture at Saqqara, to transcend the traditional mastabas by adding a new layers of symbolism to the pharaonic tomb complex. ${ }^{46}$


## Djoser's Step Pyramid at Saqqara

Moving the royal tomb to Saqqara may have created the necessity to differentiate it more clearly from the mastabas of the high officials. Before that, the mythical status of Abydos had been enough to demonstrate the pharaoh's heritage, but when the architectural environment at Saqqara is considered it is understandable that the pharaoh may have wished to express his unique status more visibly. He was the one who performed the ritual activities that unified and protected Egypt. Djoser, most likely with his architect Imhotep, and certainly with a large group of supporting artisans, overseers, and teams of construction workers, created a new type of monument at Saqqara: a giant mastaba that was eventually extended upwards and outwards to become the first great pyramid. This step forward was motivated by several factors that intersected at the time: political and social status-related issues, technical advances, and topographical and geological considerations. These factors all played a part in shaping the new monument. Together they inspired significant changes in the traditional artistic schema during the important transitional period at the start of the Old Kingdom. ${ }^{47}$

An enormous and continuous perimeter wall, often referred to as a temenos wall, was built around the Step Pyramid and served to defend and define a sacred compound. It replaced an earlier, smaller, and simpler structure, and in its final form ${ }^{48}$ the wall was a work of art in its own right.

[^5]Its four external sides were adorned with the traditional pattern of stone-built niches; the 'palace façade' motif, so closely associated with pharaonic rule at that time. Egyptologist Miroslav Verner gave the external dimensions of the enclosure wall as 544.9 m X 277.6 m , with the height of the walls being $10.5 \mathrm{~m} .{ }^{49}$ Kemp gave $545 \mathrm{~m} \times 278 \mathrm{~m} .{ }^{50}$ In the original excavation survey report the French architect J.P Lauer measured the temenos perimeter wall length to be 544.8 m by 276.85 $\mathrm{m},{ }^{51}$ but in 1960 he used slightly different values of $544 \mathrm{~m} \times 277 \mathrm{~m}$. These different sources give total perimeter lengths as variously:

| 1642 m | Lauer 1960: 2 |
| :--- | :--- |
| 1643.3 m | Lauer 1931: 60 |
| 1645 m | Verner 2003: 461 |
| 1646 m | Kemp 2005: 103 |

From these figures, it may look as if the wall was getting slightly longer over time, but this seems to be due to repeated small errors from rounding and copying of data, and repeated conversion between different units of measurement.

This perimeter length is very closely equivalent to the circumference of a circle of diameter 1000 cubits, ${ }^{52}$ which measures 3,142 cubits or 1644 m , to a very high degree of accuracy (better than $\pm$ $0.2 \%)$. The arithmetical mean of the measured perimeter values is 1644.07 m , when quoted to an accuracy of 1 cm (Figure 3-1), so the relationship is obviously very close.

In light of the accuracy of this perimeter value and the supporting architectural and iconographic evidence that follows, it is suggested that the wall's architect deliberately incorporated the properties of a circle into the final design. It was a deliberate constructional choice made by the Old Kingdom builders for symbolic reasons, adding another layer of unique symbolism over the pharaoh's tomb walls. The survey data indicates that they managed to express the concepts of circular symbolism and eternal encircling protection with impressive accuracy for such a grand scale. ${ }^{53}$

Lauer noted that the overall interior north-south length of the enclosed space was 1000 cubits, although he did not relate the perimeter of the enclosure to circular symbolism. This internal distance is the diameter of a circle that is equal in length to the outer perimeter of the wall, so the relationship between the internal length and the outer perimeter of the enclosure seems to have been the one being expressed by the architectural motif. Lauer proposed that the interior length was designed to be 1000 cubits and that the thickness of the outer walls was designed to be 20

[^6]

Fig. 3-1. Step Pyramid enclosure wall geometry (based on an image by F. Monnier).
cubits. With walls at either end of the enclosure, this gives a total length of 1040 cubits for the exterior length of the walls in the north-south direction, which converts to 544.4 m , or within half a meter of the actual value measured. ${ }^{54}$

Lauer also identified an earlier mastaba in Upper Egypt ${ }^{55}$ that may have served as a model for the

54 Lauer, "Étude sur quelques monuments de la Ille dynastie (la pyramide à degrés de Saqqarah)". The sacred walled temenos enclosure was originally smaller, however, the final structure was certainly completed deliberately to the measured dimensions and as a continuous unitary whole. The principal excavator of the site, J.P. Lauer, considered it possible that the entire final enclosure was modelled on a large Early Dynastic royal tomb from Naqada in upper Egypt that is similarly proportioned, but on a smaller scale of 10:1, so that he evidently regarded the completed Saqqara temenos to be a structure that was planned as one unified whole and which did not randomly evolve by chance. The palace façaded mastaba at Naqada he identified as a potential precursor to the Saqqara Step Pyramid complex has traditionally been attributed to Menes, who reputedly first united Egypt. In fact, it more likely dates to the time of Hor-Aha. The tomb is also now associated with the queen of Hor-Aha, Neithotep, but as it did not hold a royal burial when it was found it is also possible that it was a cenotaph, set up in a significant location. It has a substantial niched palace-façade temenos wall of 1 m in thickness surrounding it, and was rectangular in form at 54 m by 27 m . This gives a perimeter of approximately 162 m , and an internal length of close to 100 cubits; something that is echoed by the 1000-cubit internal length of the Saqqara enclosure on a larger scale of 1:10. If the circular symbolism at Saqqara had been inherited from this earlier and smaller royal mastaba at Naqada then it should have had a perimeter of close to 164 m. This equates to the circumference of a circle of diameter 100 cubits, as opposed to 1000 for the Step Pyramid temenos wall. Continuity of art forms and symbolism in this way, from the early dynasties to the third and fourth dynasties, is widely accepted. At present, the latest data from this earlier tomb suggests it may have been very slightly smaller than 164 m , at 162 m in perimeter, but it is certainly worth considering the architectural similarities that Lauer noted between this structure and the Saqqara Step Pyramid complex, with its great niched temenos wall. A.J. Spencer, Brick Architecture in Ancient Egypt (Warminster: Aris and Phillips, 1979), 149; J. Dorner, "Uberlegungen zur

Step Pyramid enclosure itself. ${ }^{56}$ In conclusion, this seems to be an example of an architectural design with a schematic element, or an emblem with symbolic relevance being used in a monumental ground plan, similar to the type described theoretically by Baines. ${ }^{57}$

At Saqqara, it seems that Djoser built a monument that embodied his own supposed ritual role; one that he believed transcended the mundane world of the officials. The new structure was built on the premise that he was a sacred leader who had inherited a divine right to rule, bestowed by his ancient ancestral god Horus. Only the pharaoh could hold the two lands together and only the pharaoh could perform the special rituals immortalized within this monument that ensured this remained the case. His ritual kingdom was separated from the mundane world outside by his encircling and protective monumental walls.

Just like the exterior architecture, the iconography developed for the interior of the Step Pyramid at Saqqara took a quantum leap forward. The type of relief scenes first displayed in the subterranean chambers were copied and used throughout the Old Kingdom and were still used in relatively unchanged forms in temple contexts 2,000 years later. ${ }^{58}$ During the Old Kingdom,


Fig. 3-2. Heb-sed relief group from the subterranean chambers below the Step Pyramid. Shown in order as arranged, from the south on the left to the north on the right (D. Lightbody).
the iconography was used very discretely for the pharaoh. It was considered special, and only his closest family members could incorporate any of the key symbols on their paraphernalia. Most notably, only the pharaoh was shown under the protection of the flying falcon Horus. At Saqqara, for the first time, Horus was shown holding the shen-ring over the pharaoh to provide him with eternal encircling protection. The flying solar disc, often referred to as Horus the Behdite, also first appeared in iconography shortly after his reign. ${ }^{59}$ It is clear that circular symbols were a notable theme within the iconographic repertoire or schema that developed during and following Djoser's reign.

None of the mastabas of the Old Kingdom officials, however, included the symbols of Horus with the shen-ring, or the heb-sed scenes. These were reserved for the pharaoh, in contrast to the palace-façade motif, which had ceased to refer only to the individual pharaoh. It is likely that this had become associated with all of the members of the pharaonic administration through its

[^7]frequent use to decorate the exteriors of the tombs of high officials.
The important new heb-sed reliefs were created in decorated corridors 33 m below the main Step Pyramid structure. Some of the chambers and corridors far below ground level are decorated with faience tiles in panels that surround the finest reliefs. They are located in a long chamber containing three scenes sunk into false doorways in the walls (Figure 3-2). The set of reliefs imitate statues or steles of the pharaoh and show him in the act of performing components of the hebsed ritual (Figure 3-3). Egyptologist and art-historian Florence Friedman noted that the scenes likely reflect rituals that were to be performed in the walled courtyard above. ${ }^{60}$ The reliefs are in doorways or alcoves surrounded by finely decorated frames (Figure 3-3). The vertical sides of the frames are adorned with columns of the pharaoh's name in palace-façade serekhs surmounted by Horus. At the corners of the frames are very prominent and rather large, round, shen-rings (Figure 3-4). It seems plausible that these frames allude to the palace façade walls encircling the ceremonial courtyard above. The symbolic encircling motif incorporated into the dimensions of the massive walls surrounding the heb-sed arena, and the complex as a whole above ground, was being referenced by these prominent round shen-rings at the corners of the decorated frames.

The three alcoves or false doors are aligned along a short corridor or elongated chamber running north-south under the main pyramid. At the south of the main enclosure near the southern wall is another deep shaft that leads down into a second, smaller, burial chamber known as the "south tomb", again around 33 m below ground level. The purpose of this structure remains unclear, but a short corridor near this "south tomb" contains three more false sunken doorways containing reliefs showing the pharaoh performing components of the heb-sed ritual. All six of these scenes below ground (Figure 3-2) are carved in low raised relief and show the pharaoh performing various ceremonies or various parts of a single ceremony. Survey analysis of the monument carried out by Friedman ${ }^{61}$ has demonstrated that these two short corridors are aligned with each other, despite the fact they are almost 200 m apart, and it is likely that all six scenes were intended to relate to one extended ritual that included several components.

This sed festival ritual, or heb-sed, was the most ancient and fundamental of all the pharaonic rituals. In this ceremony, the pharaoh ran around northern and southern bollards representing the northern and southern extents of Egypt. In this way, he would symbolically unite Upper and Lower Egypt, demonstrate his fitness to achieve this feat, and take possession of the territory he had encircled. Two huge bollards were placed at the northern and southern ends of the open courtyard above. Their form resembles the icons shown in the reliefs positioned behind the feet of the pharaoh, and it is thought that they represent the extents of Egypt. It seems that the whole complex, above and below ground, was intended to represent or be used for the ritual unification of Egypt, either in reality or symbolically in the afterlife. Friedman notes that the corridors are also aligned with a false door designed into the southern face of the main enclosure wall, and she suspects that the pharaoh was also understood to leave the complex symbolically through this door, in order to encircle the walls outside, before returning into the enclosure through that same door in the southern wall.

Given the ritual importance of the north-south axis of the complex, the round 1000-cubit internal length makes sense, as does the encircled perimeter, in light of the extension of the rituals out and around the exterior of the complex. The geometry of the building reflected the structure of the rituals to be held there.

60 F.D. Friedman and F. Friedman, "The Underground Relief Panels of King Djoser at the Step Pyramid Complex", JARCE 32 (1995), 18.

61 Ibid.


Stars on the ceilings of the burial chambers of the Step Pyramid complex suggest that the underground chambers were also associated with the heavens above. The underground reliefs included signs representing two half-sky glyphs attached to shen-rings. These can be seen on the right sides of each of the six heb-sed scenes (Figure 3-2), and they probably indicate that the pharaoh was also expected to encircle the two skies of Egypt in order to unify them. The scenes represent the rites that Djoser had to carry out for eternity, in the world of the afterlife above in the sky (pet/Nut) and below in the ground (Nun/Geb). The cardinal alignment of the whole complex, whose north-south axis deviates only a few degrees from true north, also indicates a desire to align and associate the complex, and by extension the pharaoh who was to be buried there, with the eternal heavens above.

The reliefs in the subterranean chambers depict the shen-ring carried by Horus for the first time. Djoser's patron ancestral god from Hierakonpolis flies above the figure of the pharaoh. From that time on, the shen was a prominent component of Old Kingdom heb-sed reliefs and from then on it was shown in conjunction with the repertoire of symbols that represented the pharaoh in these important ritual reliefs. This 'schema' of ritual artworks consolidated into a new tradition during Djoser's reign. ${ }^{62}$ The heb-sed had already been represented in previous dynasties, ${ }^{63}$ but in the Step Pyramid, it was shown in a new arrangement, incorporating the shen-rings for the first time.

62 E.H. Gombrich, Art and Illusion. A Study in the Psychology of Pictorial Representation (Princeton: Princeton University Press, 1972).

63 Scenes depicting the heb-sed first appeared during the Early Dynastic Period. The Narmer Macehead, recovered alongside the Narmer Palette under the dynastic era temple at Hierakonpolis, depicts the pharaoh performing the heb-sed ritual, and the arrangement is clearly a precursor of the scenes found in Old Kingdom pyramids. This artifact was probably made in late dynasty " 0 " or the early first dynasty, and it shows that key motifs of the early state such as the serekh, Horus, and the hebsed ritual, were in place at the very start of the dynastic era and were certainly associated with Hierakonpolis. An ivory tag belonging to the pharaoh Den of the first dynasty also depicts the heb-sed ritual, but the shen-ring was never incorporated in the arrangements until they were included in the reliefs in Djoser's Step Pyramid that were produced during the third dynasty. Depictions clearly showing the ritual have been found in monuments belonging to several Old Kingdom pharaohs, including in Djoser's Step Pyramid, in the valley temple of Snefru's Bent Pyramid, on fragments from the 4th dynasty temples of Khufu, in the 5th dynasty pyramid of Sahure, in the 5th dynasty sun temple of Niuserre, and in the 6th dynasty pyramid complex of Pepi II Neferkare.


Fig. 3-4. Step Pyramid frame relief corner shen-rings. Note use of double crown signifying unified Upper and Lower Egypt (D. Lightbody).

The actual pyramid within the complex at Saqqara is highly irregular in shape and plan. The base is not square, but rectangular, and the steps are sloped. There was apparently no really systematic geometry to its form beyond a gradual reduction in size of each step towards the top. The step form is thought by some researchers to have evolved or developed from the idea of building a sequence of earlier 'mastaba' type flat-top tombs on top of each other, and the lower layer of the Step Pyramid was also expanded outwards several times. It seems then that the symbolic circular proportions were, at that time, manifested only in the enclosure wall, which was more carefully built and completed.

The evidence that follows from subsequent reigns suggests that the encircling symbolism eventually migrated to the pyramid superstructures, but only after a period of experimentation at the end of the third dynasty. After Djoser's reign, several pharaonic tomb structures were commenced but not completed, and it was not until the reign of Snefru at the start of the fourth dynasty that a more controlled and regular pyramidal form was achieved.

During the intervening period at the end of the third dynasty, the first cartouches were developed and used to contain pharaonic names. This did not happen during Djoser's reign, but the symbol became the principal sign of the pharaonic ruler in the reigns that followed, eventually overshadowing even the ancient serekh. The cartouche was a modified shen-ring, elongated to incorporate the pharaoh's prenomen, throne name, or nomen, birth name, in hieroglyphs. ${ }^{64}$ Both the ring and the oval were usually depicted as a double loop of rope, tied at the base into a crosspiece formed by two flattened loops.

The first cartouche known, although very fragmentary, appears on a seal impression found in Tomb T2 at Beit Khallaf. ${ }^{65}$ It is dated to the reign of the third dynasty pharaoh Sanakht. ${ }^{66}$ Attempts to prove that the cartouche appeared earlier, in the second dynasty, are considered dubious. ${ }^{67}$ The cartouche resembles the ovals used to signify estates and built enclosures, commonly found on seals and their clay impressions, but its form is more abstract. Like the shen-ring, the cartouche served as an emblem of the type described by Baines, rather than as a direct representation of a physical enclosure or any other real tool or structure. It represented the concept of ritual encirclement, protection, and territorial dominion rather than any specific encircled or encircling monument.

The cartouche was certainly present during the reigns of rulers at the end of the third dynasty including Huni, ${ }^{68}$ but it is difficult to identify any associations between this iconography and the rather confusing and incomplete royal mortuary architecture from the late third dynasty. It is proposed here, however, that the adoption of the cartouche as the principal sign of the pharaoh at that time was related to the same ritual and political considerations that shaped the new cult architecture created during Djoser's reign.

It is proposed here that the changes in the royal mortuary architecture were motivated by political concerns, and a similar line of reasoning can also explain changes made to the system of graphical conventions used for naming and signifying the pharaoh. The process of adopting the cartouche to carry the pharaoh's name may have followed a similar logic to that which led to changes in the pharaoh's tomb architecture. With the cartouche, the pharaoh was creating a new sign that acted as a new layer of symbolism. This new symbol related to the serekh in the same way that the new pyramids related to the palace-façade mastabas. In both cases, the pharaoh deliberately differentiated himself from the rest of his officials and showed that he had transcended their mundane world by associating himself with a more sacred world, of ritually encircled eternal life. He was the one destined to carry out the never-ending encircling rituals necessary to keep Egypt protected and unified. The almost-simultaneous changes ${ }^{69}$ made in two different mediums of representation; architecture and iconography, served to re-enforce this new message. The cartouche was, therefore, chosen because it represented the encircling ritual symbolism of the shen, which had been incorporated into the pharaoh's sacred architecture at Saqqara. This new layer of symbolism was discretely associated with the pharaoh. ${ }^{70}$ The mastabas, with their palace facades, and the serekhs, had come to represent the pharaonic administration as a whole rather than the pharaoh or his closest family group. It should be noted that the serekh carried the pharaoh's own name in hieroglyphs, but the vast majority of the people in Egypt were at that time unable to read.

A new symbol that was only used for the pharaoh and only represented the pharaoh was, therefore, required.

It could also be argued that the symbol of Horus already acted in this way, but while Horus did specifically represent the pharaonic lineage that originated in Hierakonpolis, it would have been impractical to add a personal prenomen inside the graphical form of the Horus falcon. The solution seems to have been to devise a new symbol that was associated with Horus and the encircling

65 Garstang Museum, Liverpool, E5251.
66 R.H. Wilkinson, Symbol and Magic in Egyptian Art, 207; J. Garstang, Mahasna and Bet Khallaf (London: Bernard Quaritch, 1902), pl. 19, no. 7.
67 Bolshakov, 180.
68 Quirke, 123. A granite block with the cartouche of Huni was found at Elephantine.
69 At least from a modern-day perspective.
70 Spieser, Les Noms du Pharaon: Comme Etres Autonomes au Nouvel Empire, 22, refers to the cartouche as a proxy for the body of the pharaoh, and that it could function as an autonomous stand-alone symbol of the ruler without the need for an anthropomorphic representation, 33.


Fig. 3-5. Statue of stolist (priest) Ankh with shen collar (D. Lightbody).
pharaonic rituals, but which could also contain the pharaoh's personal name. That symbol was the shen-ring, already used at Saqqara within the relevant architectural and ritual contexts, and where it was already associated with Horus. When extended into a form more appropriate for containing written symbols, it formed the cartouche.

As will be discussed in the pages that follow, there was also a more profound symbolic link that related the shen-ring directly to the falcon. This was (and still is) the natural gyring flight patterns of falcons and hawks as they circle over the necropolises on the western bank of the Nile.

Additional third dynasty evidence relating to the shen-ring includes two statues that have been recovered and are now in European museums. They depict a priest who may have been responsible for carrying out special rituals in some way involving the shen and cartouche symbols that were directly related to the cult of the pharaoh (Figure 3-5). The priest was called Ankh and a statue of him in the Louvre in a seated position has a large shen symbol around its neck, like a thin pectoral collar. ${ }^{71}$ An inscription on the statue reads "Stolist (priest) of Horus, overseer/fashioner of the 3 ms -scepter, Ankh". Horus was first closely associated with the shen-ring during the period that this statue was made, probably during the reign of Djoser. Another statue of Ankh, now in Leiden, ${ }^{72}$ most likely represents the same individual and refers to him as an official or protector of the city of Nekhen, i.e., Hierakonpolis. He appears then to have held a special office or been nominated to perform a special ritual function, perhaps associated with the shen-ring, as well as Horus, and perhaps related to his ancestral home town. A depiction of Snefru's son, and Khufu's brother, Iynefer, shows him wearing a similar device around his neck. ${ }^{73}$ Both individuals adorned

[^8]with this symbol, and others who carried the title 'protector of Nekhen', ${ }^{74}$ were of high rank within the Old Kingdom court and were close to the pharaoh. They also carried the title of h3ty-r or "count", which is another high-ranking title. Panther skins with toggles also worn by both the individuals wearing the shen-ring collar indicates that they were Sem priests, intimately associated with the rituals of rebirth. ${ }^{75}$

Once the pharaoh died, his followers no longer used his personal serekh, but continued to use his own cartouche to refer to him. This indicates that the cartouche was associated with the eternal sacred nature of the pharaoh, as opposed to the mundane temporal administrative role that was the domain of the serekh. The two symbols, nevertheless, seem to have been designed to work in conjunction with each other during the pharaoh's lifetime, rather than in opposition.

[^9]74 Ankh's title of mniw/z3w Nh̆n is not unique. Other title-holders from the Old Kingdom include W3š-Ptḥ:Izi, N(y)-ḥtp-Ptḥ, Nb-k3w-Ḥr:Idu, 'nȟ-m- ${ }^{〔}-H$ Hr:Zzi, and Wr-ir.n.i.
75 M. Eaton-Krauss, "Two Masterpieces of Early Egyptian Statuary", OMRO 77 (1997), 11.

## Fourth Dynasty

## Snefru's Meidum Pyramid

During the reign of Snefru (Figure 4-1), the first true pyramid was constructed at Meidum. Although it was perhaps begun by the previous pharaoh Huni, Snefru most likely completed the pyramid around 2,600 B.C. In its final construction phase, Snefru turned it from a step pyramid into what may have been the first true pyramid. This was accomplished towards the end of his reign after changes had been made to its design. He also completed two other great pyramids, at Dahshur. The Meidum pyramid was completed around the same time that changes were made to the Bent Pyramid at Dahshur south. ${ }^{76}$ Snefru oversaw several leaps forward in pyramid engineering, some of them probably in response to structural problems encountered due to the increased size of the monuments being erected, ${ }^{, 77}$ but some aspects of the developments involved more symbolic and ritual concepts associated with the afterlife of the pharaoh.


Fig. 4-1. Snefru's cartouche and serekh can be translated as "he has perfected me", and "lord of ma'at" (D. Lightbody).

By filling in the steps at Meidum with limestone encasing blocks, Snefru created the pyramidal form that is so familiar to us today. The Meidum pyramid, however, either collapsed and/or was dismantled at some point after it was completed, and is sometimes known as the Collapsed Pyramid or the Failed Pyramid as a result. Petrie was the first to accurately reconstruct its final and original dimensions during a survey he carried out in the 1890s. He excavated the complex and carried out surveys of other monuments in the Meidum area. ${ }^{78}$

What Petrie found at Meidum was evidence that the original 'as-built' proportions of the Meidum pyramid, in its final form, were such that the perimeter of the base equaled the circumference of a circle produced by using the pyramid's height as a radius. This relationship only holds for pyramids with the precise side slope found at Meidum (Figure 4-2).

Surveys since then have confirmed these proportions. It also appears that the actual numerical values of the dimensions used in cubits have some archaeological value that aids architectural interpretation. They provide information about the intentions and choices made by the Old Kingdom pyramid architects, as well as information about the technical systems they used.

The values Petrie calculated for the dimensions of the completed structure were as follows:
As-built height 92 m , pyramid side lengths 144 m , pyramid base perimeter $576 \mathrm{~m} .{ }^{79}$ In Old Kingdom

[^10]

Fig. 4-2. Meidum pyramid's exterior geometry when completed (D. Lightbody).

Fig. 4-3. Bent Pyramid's mortuary temple with its two steles reconstructed (F. Monnier).

royal cubits this equates to 1100 cubits around by 175 cubits tall. This produced a pyramid with the relevant symbolic circular proportions. The dimensions were of a size appropriate to the technical development taking place during the transitional phase at the end of the third dynasty and start of the fourth. ${ }^{80}$ Based on the sequence of pyramid building projects carried out during Snefru's reign, it does not seem that this final form was the initial intended choice of dimensions or slope for the Meidum pyramid. It may only have been after experimentation with pyramid slopes and geometries
elsewhere that this relationship was proposed for use at Meidum. The particular slope may have been noted as potentially symbolic during that final design process. The encircling architectural motif already employed at Saqqara in the temenos wall, was adapted and applied to the pyramidal form for the first time at Meidum.

Fig. 4-4. Bent Pyramid's only recovered stele (F. Monnier).


The completed pyramid complex at Meidum had its own temenos wall containing subsidiary buildings, but these were greatly reduced in relative size when compared to Djoser's complex. It seems that at Meidum, most of the monumental emphasis was transferred to the pyramid itself. At Djoser's Step Pyramid the circular proportions had been applied to the temenos wall, and not to the irregularly shaped Step Pyramid, but it seems that at Meidum the proportions, and hence the architectural motif, were applied directly to the true pyramid for the first time.

Petrie noted these special proportions in the excavation report for Meidum (which he referred to
as Medum, , ${ }^{81}$ and he concluded that the pyramid was indeed designed so that the base perimeter was the length of the circumference of a circle with a radius equal to the pyramid's height. In three dimensions, this produces the now familiar side slope angle of $51.84^{\circ}$ in decimal degrees (or $51^{\circ} 50^{\prime} 35^{\prime \prime}$ in degrees proper)..$^{82}$ The Egyptians themselves would probably have expressed this slope as a seked of $51 / 2$ palms. ${ }^{83}$

Although the Meidum pyramid was built before the Great Pyramid, Petrie surveyed it after he had surveyed at Giza. This means that the older pyramid at Meidum confirmed the conclusions regarding symbolism that he had already drawn from his survey of the Great Pyramid at Giza, and he held this architectural relationship to be a fact for the rest of his life. ${ }^{84}$


Fig. 4-6. Iconographic arrangement from furniture belonging to Hetepheres. One of the earliest uses of the flying solar disk, and the standard text expressing encircling protection (D. Lightbody).

The iconography from Snefru's reign provides powerful evidence for a continuity of traditions from Djoser's reign into Snefru's. Fragments of reliefs recovered from the Bent Pyramid's valley temple show many elements of heb-sed scenes ${ }^{85}$ including examples showing Horus grasping shen-rings. Half-sky glyphs surmounted by shens also appear, but the inclusion of cartouches ${ }^{86}$ within the heb-sed arrangements was new, and a significant difference with respect to the earlier versions found in Djoser's Step Pyramid. There were no relief scenes within the burial chambers of Snefru's pyramids, but pairs of magnificent steles were erected at the small mortuary temples built outside his pyramids, on the east side facing the Nile (Figure 4-3). The steles at Meidum were never decorated, perhaps because the pyramid partially collapsed before they were completed, but

81 Ibid., 6. "This angle, it will be seen, is just that of the Great Pyramid of Gizeh, which was built next after this pyramid. And we have therefore to consider if any of the theories concerning the size of that are elucidated by this. Now the most simple and promising theory is that the ratio of $7: 44$, for that of a radius to a circumference, is embodied by the Great Pyramid height being $7 \times 40$ cubits and its circuit $44 \times 40$ cubits; in short, that it was built 7 and 44 times a modulus of 40 cubits. The angle being the same here at Medum the ratio $7: 44$ will of course hold good; the question is if a simple modulus was used here also. The base being 5682.0 inches, it is $7 \times 25$ cubits in height, and $44 \times 25$ cubits in circuit ; the cubit required being 20.66 +/- .01 inches, or varying from 20.63 to 20.70 according to different sides, which is just the usual range of varieties of the Egyptian cubit. We see then that there is an exactly analogous theory for the dimensions of Medum to that for the Great Pyramid ; in each the approximate ratio of $7: 44$ is adopted, as referred to the radius and circle ; in the earlier pyramid [Medum] a modulus of 25 cubits is multiplied by these numbers to fix the dimensions ; in the later pyramid [the Giza Great] a modulus of 40 cubits is used".
82 M. Lehner, The Complete Pyramids (London: Thames \& Hudson Ltd, 1997), 17.
83 J.A.R. Legon, "The 14 to 11 Proportion at Meydum", DE 17 (1990).
84 W.M.F. Petrie, Wisdom of the Egyptians, British School of Archaeology in Egypt and Egyptian Research Account ; [Vol. LXIII] (London: British School of Archaeology in Egypt and B. Quaritch Ltd, 1940), 30. This publication is now available on the Internet Archive website in both scanned and digital forms. See https://archive.org.
85 A. Fakhry, The Monuments of Sneferu at Dahshur. Volume 2. Part 1. The Temple Reliefs (Cairo: General Organization for Government Printing Offices, 1961).
86 Ibid., 144, fig. 91.


Fig. 4-7. Great Pyramid's exterior geometry showing its circle-related proportions, and below, diagrams showing how these relate to the cubit and the cubit's own relationship with the circle (D. Lightbody).
one of the Bent Pyramid's two steles was recovered and the iconography is revealing. ${ }^{87}$ The steles seem to have consisted of a pair of large serekhs, each surmounted by a figure of Horus (Figure 4-4), and most likely arranged with the two falcons facing each other. On the recovered left side stele the pharaoh is shown sitting within the serekh wearing his heb-sed robes alongside his titulary including a large cartouche. The Horus falcon standing on top of the serekh is in classic standing position, ${ }^{88}$ and while its head is now lost, it seems likely that it wore a white crown as it was on the southern stele and represented Upper Egypt. The falcon on the right stele would have worn a red crown representing northern Lower Egypt (Figure 4-5). Equivalent symmetrical arrangements have been found in later

[^11]monumental relief scenes outside Old Kingdom pyramids (Figure 5.2), ${ }^{89}$ and the arrangements signified that the two regions of Egypt were ritually united and protected by the pharaoh.

If the cartouche was indeed closely related symbolically to the pyramid architecture at that time, then placing it within the serekh motif, which was itself related to the palace façade architecture of the temenos walls, was meaningful. If similar iconography was intended to be placed on the steles at Meidum that were never decorated, then the cartouche within the palace façade serekh would have been additionally meaningful, considering that the encircling motif integrated with that pyramid's architecture was contained within a temenos wall.

Early fourth dynasty pharaonic iconography has also been recovered from the funerary assemblage of Snefru's wife, Hetepheres. Her personal funerary items were found stashed in a tomb shaft at Giza on the east side of her son Khufu's pyramid. ${ }^{90}$ A pair of scenes on the ends of her finely decorated curtain box included the earliest-known version of the flying solar disc and a scene showing the vulture goddess Nekhbet holding the shen-ring above the pharaoh. Under the solar disc is a cartouche of Snefru and to the left side of it is a set of glyphs spelling out the phrase "protection surrounding, life forever" (Figure 4-6).

## s3 $h 3$ 「nh $d \underline{d}$

This became a classic phrase often shown in relief scenes written directly behind the pharaoh. As discussed by Edward Brovarski, the word $h 3$ in this context in fact means "around and behind". ${ }^{91}$ The text, therefore, supports the conclusions drawn from the iconography, that these scenes referred to the encircling protection of the pharaoh that was an important ritual concept developing at the time.

## Khufu's Great Pyramid

After the end of Snefru's reign, and only a few years after the Meidum pyramid had been completed, the pharaoh Khufu began building his pyramid at Giza using the same special proportions that had been used at Meidum. The "Great Pyramid" still standing west of Cairo in Egypt was the first and foremost of the giant Giza trio and was the largest and most carefully built of any pyramid in Egypt. Like at Meidum and Saqqara, the Great Pyramid had a large temenos wall, but much less emphasis, relatively speaking, was placed on its construction. The wall was made of mud brick and was not apparently niched with the palace façade motif at all. This abandonment of the palace-façade motif surrounding the pharaoh's tomb ${ }^{92}$ seems to fit within a hypothesis whereby the pharaoh was motivated to elevate his own status above his contemporaries in the administration. Greater emphasis was placed on the stone pyramid itself, a form only used by the pharaohs and their wives at that time. Less emphasis was placed on incorporating the palace façade motif.

Due to its size and the survival of some well-preserved casing stones around its base, it has been possible to reconstruct the Great Pyramid's original height and base/side lengths, and hence its overall proportions, with a high degree of accuracy and confidence. The details of the pyramid's

89 L. Borchardt, Das Grabdenkmal Des Königs S'ahu-Re (Band 1): Der Bau (Leipzig: Hinrichs, 1910), 44, 45.
90 Dynasty 4 curtain box. Cairo JE 72030. Belonged to Hetepheres. Found in tomb G7000X at Giza. See G.A. Reisner and W.S. Smith, A History of the Giza Necropolis. Volume II. The Tomb of Hetep-Heres the Mother of Cheops: A Study of Egyptian Civilization in the Old Kingdom (Cambridge, MA: Harvard University Press, 1955), figs. 28a, 28b, 29a, 29b, pls. 11, 12.
91 E. Brovarski, "Once More Hr 'Pyramid'?", in Sitting Beside Lepsius: Studies in Honour of Jaromir Malek at the Griffith Institute, eds. D. Magee, J. Bourriau, and S. Quirke (Leuven: Peters, 2009).

92 The palace-façade motif did make a comeback in pharaonic tomb architecture during Menkaure's reign, perhaps signifying a more conciliatory attitude towards the administration.
construction have been discussed at great length elsewhere, ${ }^{93}$ but in the current study the technical developments are of interest insofar as they elucidate the symbolism incorporated within the architecture.

The actual 'as-built' side lengths of the Great Pyramid's original base are derived from surveys carried out by Petrie in $1882 .{ }^{94}$ Comparable surveys were carried out by Cole in 1925 for the Egyptian government, ${ }^{95}$ by Dorner working on his PhD in the early 1980s, ${ }^{96}$ by Lehner and Goodman in $1985,{ }^{97}$ and most recently, by Glen Dash's team working for AERA in 2015. ${ }^{98}$ They all found that the side lengths were precisely laid out, deviating by less than $+/-11 \mathrm{~cm}$ from the average over a distance of more than 230 m on each side, which is impressively consistent by any measure. The perimeter was almost a kilometer in length; at 921.38 m according to Petrie, 921.455 m according to Cole, 921.44 m according to Dorner, and 921.390 m according to Dash.

Its as-built height, reconstructed from the archaeological evidence by Petrie, and more recently by others ${ }^{99}$ is estimated to have been between 146.55 m and 146.75 m tall. These dimensions indicate that the original building was 280 cubits in height by 1760 cubits around in standard royal cubits, ${ }^{100}$ equaling 440 cubits on each side at the base level. This means that the perimeter of the base equaled the circumference of a circle with a radius equal to the pyramid's final height, to an extremely high degree of accuracy. These are exactly the same proportions used at Meidum, but expressed on an even grander scale (Figure 4-7).

It was this level of accuracy in the circular proportions of the Great Pyramid that first brought attention to the serious possibility that geometric properties had been deliberately incorporated into the primary dimensions of the monuments. The Old Kingdom cultural context summarized in the rest of this publication now seems to confirm that this was indeed the intended reality. Other important

93 D. Lightbody and F. Monnier, The Great Pyramid: Haynes Operations Manual..
94 W.M.F. Petrie, The Pyramids and Temples of Gizeh - 1st Edition (London: Field \& Tuer, 1883). N 230.363 m E $230.320 \mathrm{~m}, \mathrm{~S} 230.365 \mathrm{~m}, \mathrm{~W} 230.342 \mathrm{~m}$, giving an average of 230.348 m .
95 J.H. Cole, The Determination of the Exact Size and Orientation of the Great Pyramid of Giza, vol. Paper no 39 (Cairo: Survey of Egypt, 1925). N 230.353 m, E 230.391 m, S 230.454 m, W 230.357 m, giving an average of 230.364 m.
96 J. Dorner, Die Absteckung Und Astronomische Orientierung ägyptischer Pyramiden (Innsbruck: Universitaet Innsbruck, 1981); "Das Basisviereck Der Cheopspyramide", in Structure and Significance, Thoughts on Ancient Egyptian Architecture, ed. P. Janosi (Vienna: Verlag der Öestrreichinschen Akademie der Wissenschaften, 2005). Dorner's measurements gave side lengths of N 230.328 m , E $230.369 \mathrm{~m}, \mathrm{~S} 230.372 \mathrm{~m}, \mathrm{~W} 230.372 \mathrm{~m}$, giving an average of 230.360 m .

97 The earlier survey carried out by Lehner and Goodman is detailed in "New Angles on the Great Pyramid", AERAGRAM 13, no. 2 (2012).
98 G. Dash, "The Great Pyramid's Footprint: Results from Our 2015 Survey", AERAGRAM 16, no. 2 (2016), 11. N 230.329 m, E $230.334 \mathrm{~m}, \mathrm{~S} 230.384 \mathrm{~m}, \mathrm{~W} 230.407 \mathrm{~m}$, giving an average of 230.363 m .
99 J. Bodsworth carried out the latest height estimations based on Petrie's survey data. He wrote that Petrie's data was ideal for such an exercise as he gives so many measurements as offsets from, for instance, the surrounding stone pavement or other internal elements, and the modern reconstruction is effectively a three-dimensional version of Petrie's measurements and drawings. 'Piling up' the individual core layers was straightforward using Petrie's measurements, but putting the casing on was a different matter, as it was necessary to choose which exact measurement to use for the final height, or conversely, what precise angle to use. This is the crucial part of the reconstruction calculation. Bodsworth carried out a detailed study of all the surviving evidence, from casing stones and documentary statements regarding the casing stones. His final 3D reconstruction included his best estimation of the casing stone arrangement based on this dataset, and he was, therefore, able to calculate the final as-built height of the pyramid from this. He wrote regarding estimates of the original height: "As we don't know what tolerances the builders worked to or even the amount of settling etc. that might have happened over thousands of years, I think Petrie's stated tolerance of plus or minus 7 inches is still valid and as close as we'll ever get. Even if we knew for certain what method the Egyptians used to establish the height, we will never know how close they actually got to it". The total height he estimated is close to Petrie's 5771 to 5773 inches, $+/-7$ inches, or 146.6 m . Circular proportions based on the side lengths extrapolate to 146.58 m in height (based on an approximation of $3+1 / 7$ th for the length of a circumference of a circle diameter 1 , so that the circular symbolism proposal certainly falls accurately within any of the estimated ranges and tolerances.
100 Cubits of 0.5235 m .

Egyptologists have reached the same or similar conclusions, ${ }^{101}$ as have scientists from other domains including those studying the history of mathematics. ${ }^{102}$ Petrie's conclusion that circular proportions


Fig. 4-8. Relief from Khufu's pyramid reused in Lisht, showing Horus with the shen-ring in fine detail, most likely flying above a depiction of the pharaoh. Now in the Boston MFA (D. Lightbody).
guided the design, therefore, appears to have been correct. As he concluded: "these relations of areas and of circular ratio are so systematic that we should grant that they were in the builder's design" ${ }^{103}$

In practice, the slope would most likely have been applied to the outer Tura limestone casing during the final construction phase through the use of the 'seked' system of angle measurement, which is attested in early Middle Kingdom mathematical papyri. ${ }^{104}$ The slope-value was first calculated from the architects' choice of base and height lengths, and then converted into sekeds for application on-site using practical tools based on the 7-part cubit and probably including plumb bobs. In the case of the Great Pyramid the resulting seked would have been ' 5 palms and 2 digits', so that for every one cubit rise, the face slope retreated by 5 palms and 2 digits. This produces the observed slope precisely, ${ }^{105}$ and equates to the now famous pyramid slope angle of $51.84^{\circ}$ in decimal degrees, or $51^{\circ} 50^{\prime} 35^{\prime \prime}$ in degrees proper. ${ }^{106}$

[^12]
## The so-called King's Chamber of the Great Pyramid

The so-called "King's Chamber" of the Great Pyramid of Giza is one of the most historically significant architectural spaces in the world. Despite cracks in the massive granite roof beams and


Fig. 4-9. Khufu's King's Chamber's internal encircling geometry as determined by Petrie (D. Lightbody).
some signs of movement in places, ${ }^{107}$ the state of the walls remains good given their great age, meaning that that the chamber's original dimensions have been reconstructed with a high degree of confidence (Figure 4-9).

After surveying the chamber and analyzing its geometry in cubits, palms, and digits, ${ }^{108}$ Petrie noted that the perimeters of the northern and southern walls are equal to the length of circles formed by using the width of the chamber as a radius. The width, which runs in a north-south direction, is precisely ten cubits or 280 digits in length. ${ }^{109}$ The circuit of the northern and southern walls is 1760

107 Due to the construction material being dark rose Aswan granite, a rock-type abundantly utilized in pyramid burial chambers, and the fact that the chamber has never seriously failed structurally, it has survived in relatively good condition for its great age and in a form that seems to be very close to its original dimensions. Nevertheless, as Petrie pointed out, the roof is seriously compromised: "These openings or cracks are but the milder signs of the great injury that the whole chamber has sustained, probably by an earthquake, when every roof beam was broken across near the South side; and since which the whole of the granite ceiling (weighing some 400 tons), is upheld solely by sticking and thrusting. Not only has this wreck overtaken the chamber itself, but in every one of the spaces above it are the massive roof-beams either cracked across or torn out of the wall, more or less, at the South side; and the great Eastern and Western walls of limestone, between, and independent of which, the whole of these construction chambers are built, have sunk bodily. All these motions are yet but small-only a matter of an inch or two-but enough to wreck the theoretical strength and stability of these chambers, and to make their downfall a mere question of time and earthquakes". Petrie's earthquake hypothesis has been questioned..
108 In 2002, the author carried out a manual measurement of the King's Chamber using traditional tape measures.
109 Petrie, Wisdom of the Egyptians, 29: "This same proportion is found in the King's Chamber ; the breadth is the radius of a circle equal to the circuit of the side wall. Here, as the workmanship is very exact, and the length is exactly double the breadth, the height of the chamber is the dimension which makes up the odd amount. Keeping to the old $7: 11$, or rather $7: 44$, proportion, there being 7 palms in the cubit, the radius or chamber breadth is 70 palms, and the side circuit 440 palms. The length top and
digits. The same numerical values and circular proportions, therefore, appear to have been applied to the primary dimensions of the King's Chamber as were applied to the principal outer dimensions of the Great Pyramid. Setting the internal and external dimensions side by side allows comparison of the numerical values used as well as the proportions, and both seem to be significant, as follows:

External pyramid dimensions: 1760 cubits around, 280 cubits high equaling radius of perimeter circuit.

Internal chamber dimensions: 1760 digits for circuits of $\mathrm{N} \& \mathrm{~S}$ walls, 280 digits chamber width equaling radius of wall perimeter circuits. ${ }^{110}$

When considered within the context of Old Kingdom encircling symbolism (Figure 4-8), and within an ideology whereby the pharaoh was the one who unified northern and southern Egypt by performing encircling rituals, the incorporation of encircling geometry in the northern and southern walls of the pharaoh's burial chamber makes good sense (Figure 4-9).

The fact that these two walls are also the departure points for shafts that lead to the outer faces of the pyramid and then aim directly at the northern and southern skies also seems to be related to the iconography of the heb-sed reliefs found in Old Kingdom pyramids. ${ }^{111}$ Pairs of half-sky glyphs surmounted with shen-rings are almost always shown in those arrangements, indicating the pharaoh's ritual role as the one who would encircle the two skies and unite northern and southern Egypt in the heavens above and on the ground below. The architecture at Giza and the Old Kingdom iconography are clearly linked by these symbolic concepts, designed to elevate the pharaoh's status and manifest his role as the only one who could unify and protect Egyp. ${ }^{112}$

This red granite room is surely the finest example of Old Kingdom pharaonic architectonic ideology, petrified in monumental architecture.


Fig. 4-10. The sema-tawi motif symbolizing the unification of Upper and Lower Egypt
(D. Lightbody).

[^13]

Fig. 4-11. Relief arrangement including depiction of Khufu enthroned, from Hatnub (D. Lightbody).

## Djedefre and Kaba

These fourth dynasty pharaohs both incorporated ovoid red-granite sarcophagi into their pyramids, a form that resembles the cartouche to some extent. The fragmented nature of the rest of their monuments and the lack of clarity regarding the chronologies of their reigns makes it difficult to draw any further conclusions from this fact. Such a form would, however, fit well within a tradition where the pharaoh was protected and encircled in the afterlife. ${ }^{113}$ Examples of encircling cartouche symbolism incorporated into the designs of New Kingdom red granite pharaonic sarcophagi are discussed later in this publication, and the protective physical attributes of red granite may have been directly associated with the encircling protective solar symbolism.

## Menkaure

Thanks to the preservation and recovery of several fine statues in Menkaure's valley temple at Giza, significant examples of pharaonic iconography from his reign have survived. The sides of an enthroned travertine/Egyptian alabaster statue of the pharaoh ${ }^{114}$ that was excavated from his valley temple are decorated with a pair of scenes that include the sema-tawi motif (Figure 4-10). The sema-tawi motif was known since the Early Dynastic Period, and was used throughout Khufu's reign and later (Figure 4-11). ${ }^{115}$ It signified the unification of the two lands of Egypt into one. The seated statue of Menkaure, which is now missing its upper part, was most likely designed to sit in a chapel in the west end of his valley temple facing east, to be observed by viewers arriving from the east. The panels on the left and right sides of the seat base were, therefore, designed to represent the southern and northern parts of Egypt, respectively (Figures 4-12 and 4-13). The enthroned pharaoh was then presented at the meeting point between the two lands, and was portrayed as the one who unites Egypt. The statue was most likely also designed to be oriented and positioned with respect to the decorative relief program within the cardinally aligned temple, and by extension with respect to the other aligned monuments of the Giza plateau, and with respect to the whole of the Egyptian territory outside.

114 Now in the MFA Boston 09.202.
115 The Hatnub quarry inscribed scene includes Khufu seated on a throne bearing the sm3-t3.wy motif.


Fig. 4-12. Photographs of the pair of scenes on the sides of Menkaure's enthroned statue. On the left is the side designed to face south, on the right is the side designed to face north (D. Lightbody).


Fig. 4-13. Line drawings of the pair of scenes on the sides of Menkaure's enthroned statue including pairs of opposing falcons in flight, carrying shen-rings. Same orientation as photographs left (D. Lightbody).


Fig. 4-14. The relative positions of the three pharaonic pyramids at Giza incorporated an encircling geometry into the cardinally aligned perimeter. This would only have been established during Menkaure's reign (D. Lightbody).

Both side panels on the statue's base include serekhs and cartouches. ${ }^{116}$ Pairs of Horus falcons holding shen-rings overfly the two iconographic arrangements, and touch wingtips in a way that recalls the sky glyphs that traverse the tops of similar, simpler, scenes. ${ }^{117}$ This touching together of the wings of the two falcons elaborates the encircling unification motif, and alludes to the ritual event that was to take place on earth below and in the skies above. Another Horus falcon stands on a serekh at the left of the southern side panel wearing the white crown of Upper Egypt. A similar falcon on the northern side panel does not wear a crown, perhaps due to lack of space to draw the spiral that usually extends from the front of the red crown, which would normally appear in that arrangement.

Another fine statue uncovered in Menkaure's valley temple is a greywacke triad that shows the pharaoh flanked by two goddesses. Hathor on his right side carries a three-dimensional version of a


Fig. 4-15. Triad statue of Menkaure recovered from his valley temple. One of a series of statues designed as focal points where offerings would be made by representatives of each nome region of Egypt. This particular statue is remarkable in that it lacks a serekh and because the goddesses carry shen-rings. These differences may have alluded to the fact that the pharaoh had died by the time this statue was made, and had, therefore, entered his eternal, encircled, resting place, leaving the secular world associated with the serekh behind. In this afterlife context, Anubis, jackal god of the 17th Cynopolis nome, was a suitable companion (K. Moyls).

[^14]shen-ring, represented as a tied coil of rope in her right hand. ${ }^{118}$ On his left side is a personification of the $17^{\text {th }}$ nome region, Cynopolis, signified by the appropriate standard on her head, which includes a recumbent Anubis jackal facing left towards the pharaoh. This goddess also holds a shen-ring, but in her left hand. These two shen-rings appear to be the only three-dimensional representations of shen-rings known from the Old Kingdom, and there are other remarkable aspects to this statue. ${ }^{119}$ None of the figures represented in the other triad and dyad statues recovered from this valley temple carry shen-rings.

With respect to Menkaure's pyramid, the precise dimensions, proportions, and side slopes have proved difficult to measure as a result of the monument's partially ruined and dismantled exterior state, but the available information indicates that its proportions (as opposed to its dimensions) were close of those of the Great Pyramid. ${ }^{120}$ More interestingly in this case is the proposal that the monument's final position relative to the other two pharaonic pyramids at Giza was chosen so that the overall ground plan covering the Giza triplet, as a group, encompassed and incorporated the traditional encircling symbolism (Figure 4-14). An impressive and powerfully simple geometry appears to have been included into the design of the plateau's architecture, as it evolved towards completion at the end of the fourth dynasty. ${ }^{121}$

Once Menkaure's architects had positioned the square ground plan of his pyramid, the perimeter length of the rectangular plan surrounding the three great pharaonic tombs on the Giza plateau was precisely equal to the circumference of a circle radius 1000 cubits. It was also, therefore, precisely twice the perimeter of the great wall surrounding the Step Pyramid complex at Saqqara. ${ }^{122}$

118 The triad statue of Menkaure flanked by Hathor and a Cynopolis nome goddess with jackal insignia over her head, now in Cairo JE 40679, was discovered in Menkaure's valley temple by Reisner's team.
119 It is notable that this triad statue with shen-rings is the only triad that does not include a serekh in the base inscriptions. Friedman considers that this indicates that this triad was completed after the death of the pharaoh. The shen-rings carried by the goddesses may then signify that the deceased pharaoh had by then entered his eternal, encircled, resting place. The jackal Anubis, emblem of the $17^{\text {th }}$ Cynopolis nome, was a suitable partner for the pharaoh on this afterlife journey. See F.D. Friedman, "Reading the Menkaure Triads, Part 2 (Multi-Directionality)", in Old Kingdom, New Perspectives. Egyptian Art and Archaeology 2750-2150 BC, eds. N. Strudwick and H. Strudwick (Oxford: Oxbow Books, 2011), 111.
120 Verner, 463.
121 J.A.R. Legon, "The Plan of the Giza Pyramids", Archaeological Reports of the Archaeology Society of Staten Island 10, no. 1 (New York: AIA, 1979); idem, "The Giza Site Plan Revisited", GM 124 (1991); idem, "On Pyramid Dimensions and Proportions", DE 20 (1991); idem, "A Ground Plan at Giza", DE 10 (1988); idem, "The Giza Ground Plan and Sphinx", DE 14 (1989). In cubits, Legon found that the north-south length of the rectangle shown in the diagram (Figure 4-14) expresses the value of root $3 \times 1000$ royal cubits exactly, while the east-west width of the rectangle is very slightly in excess of root $2 \times 1000$ royal cubits. The exact figures he calculated from the theodolite surveys were as follows: Distance N-S =1731.97 cubits; Distance $\mathrm{E}-\mathrm{W}=1417.42$ cubits. These side lengths, if they were intended to represent a theoretically ideal rectangle of root 2 x root 3 thousand cubits ( $1732 \times 1417.5$ cubits), were only $0.002 \%$ and $0.2 \%$ in error. Square roots of 2 and 3 would certainly have been useful and perhaps widely used in ancient Egypt, as they constitute the side lengths of squares of areas 2 and 3 square units. This would have been useful for doubling or tripling field areas, and there is evidence that a unit called the doubleremen was effectively a length of root 2 cubits and was used for that purpose. Why would the ancient Egyptians have used these root values in particular? Was there a symbolic aspect to square roots as well as for circular proportions? The answer to this question reveals a crucial aspect of these square roots that is unknown in the modern world today, where only 'absolute' relationships are considered of use or of importance, c.f. T.E. Rihill, Greek Science (Oxford: Oxford University Press, 1999), 44. Close approximations are not used, or at least are not taught in mathematics classes, as they are not considered 'high mathematics'. Proofs and pure mathematical relationships are now taught almost to the exclusion of older rules of thumb that were handy for practical reasons. Nevertheless, the cultural context of ancient Egypt indicates that practical building and construction was the priority, so that accurate rule-of-thumb relationships such as is described below would have been considered useful and perhaps important. The 'forgotten' practical relationship that was included here is that root 2 plus root 3 equals 3.1462 , or almost exactly the length of the circumference of a circle with a diameter of 1 cubit. This means that the perimeter length of the rectangle surrounding the three great pharaonic tombs at Giza is $2 \times \pi$ thousand cubits in length, and so again it seems that the circular symbolism was being applied around the royal tombs, in a novel manner.
122 This Giza plan perimeter, with its circular symbolism, is precisely double the perimeter of the Saqqara temenos wall (+0.2\%), so that whereas Saqqara's wall represents the perimeter of a circle diameter 1000 cubits, Giza's plan represents a larger circle, with a radius of 1000 cubits.

It seems that Menkaure's architects had drawn inspiration from the old symbolism used in the Step Pyramid complex.

The double Horus falcons holding shen-rings, overflying the unification scenes on Menkaure's throne, should then perhaps be interpreted in light of this double-sized, encircling, architectural "schematic" statement. Similarly, the two shen-rings carried by the goddesses on the triad statue discussed above could plausibly allude to this double-encircling protective motif surrounding the Giza necropolis (Figure 4-15).

The scale and sophistication of the architectural symbolism is perhaps disconcerting at first glance, however, when contextualized within a monumental and ritual tradition that stretched back more than a century, at least to the time of Djoser, it begins to make good sense.

It seems most likely that this relationship was deliberately included into the design of the plateau, but only when the last of the three great pharaonic pyramids was built at Giza. Despite an appearance of organized planning at Giza, evidence from the other pharaonic tombs built during the fourth dynasty does not support a proposal where the final layout was pre-planned during Khufu's reign. Multi-generational planning was not typical of the Old Kingdom pharaonic culture, and the majority of the design work was based on retrospective considerations; on the existing monuments already erected by predecessors. What Menkaure's architects included was most likely an additional layer of the traditional symbolism, adding to the power of the pre-existing Giza site. This encircling perimeter on the landscape was included when the position of Menkaure's tomb was being finalized. Based on this logic, it was, therefore, Menkaure's architects who were responsible for creating the circular symbolism within the ground plan encompassing the three Giza pyramids.

Following the proposal that the cartouches and the pyramid monuments were closely related, Menkaure's prenomen, which translates approximately as 'established are the ka-souls of Re' or 'the ka-souls will endure like Re' may have had special significance. The use of the plural form of 'ka' represented by three ka symbols in his cartouche is unusual, as the pharaoh himself was


Fig. 4-16. Menkaure's cartouche (D. Lightbody).
only thought to have one ka (Figure 4-16). It seems reasonable to consider this choice in light of the possible inclusion of the pyramids of his pharaonic ancestors within the extended encircling design of his own eternal resting place. Those earlier pyramids were built for the ka-souls of his grandfather Khufu and father Khafre. Like that ground plan encircling the trio of pharaonic pyramids, Menkaure's cartouche encircles the hieroglyphs for ka-souls. The significance of the circular Re sign in this context would also be especially meaningful.

The positions of the pyramids and overall layout of the pharaonic necropolis nevertheless had to be appropriate to the landscape, which limited the range of choices available to the builders.

Other factors would have determined the positions of the pyramids to some extent, such as the topography of the landscape running along the horizon of the western desert, and its visibility from the Nile floodplain. The availability of stone, proximity to Memphis, and the courses of the Nile's channels, which would have been used to transport materials, were also factors that would have been considered when planning a new pyramid. ${ }^{123}$

The three giant pharaonic pyramids at Giza are built on the eastern side of a raised slope called the Mokattam Formation. Parts of the formation formed a useful and relatively flat rocky platform at the eastern edge of the plateau, and parts of it were deliberately levelled. The natural outcrops and slopes would have been the primary factors determining pyramid location choice, but within those geological constraints, the pharaoh's architects would have been free to choose particular locations and to measure out particular ground plans that incorporated symbolic considerations on a grand scale. ${ }^{124}$

[^15]124 Other symbolic traditions, such as those associated with the pole stars and the rising and setting sun may have been influential, and may have created the necessity for permanently clear lines of sight to the skies from the monuments to the north, east, south and west. For religious reasons related to the ascension of the pharaoh's spirit to the heavens, these directions were all considered to point at important regions of the sky. Aesthetic considerations may also have ensured that the great monuments were built in locations that were visible from the inhabited parts of the Nile floodplain, such as around Memphis and the apex of the Delta, and from the main pre-existing temples in those areas.

## Fifth Dynasty

The last pharaoh of the fourth dynasty, Shepseskhaf, appears to have rejected the Giza plateau in favor of a more prudent mastaba-style burial at Saqqara. The reasons for that retroactive move are still under discussion, but from then on the pharaonic tomb monuments of the Old Kingdom were greatly reduced in scale. This unfortunately means that the exterior structures of the smaller


Fig. 5-1. Userkhaf relief reused at Lisht (D. Lightbody).
pyramids and temples of the later Old Kingdom are mostly in a ruinous state, and it is difficult to obtain accurate data regarding the dimensions of original ground plans or the angles of sloping faces. The architectural data is not adequate to allow conclusions to be drawn from metrical analysis alone. On the other hand, fifth dynasty mortuary architecture was notable for increasing numbers of finely carved stone reliefs carrying the schema or repertoire of pharaonic iconography. These reliefs were created on temple walls, on temple columns, on stone statuary, and similar designs were carved on fine stone vessels used in those places. At the end of the fifth dynasty, the ritual mortuary concepts were first expressed on the walls of the tomb chambers in the form of the Pyramid Texts.

Fine reliefs were incorporated into royal mortuary temples that were larger in scale relative to the tombs, and of increased complexity, when compared to the monuments of previous dynasties. The temples contained more graphical representations and more hieroglyphic texts. Overall, it seems that more effort was spent on producing fine iconography and usable cult spaces rather than creating massive, overbearing, inaccessible, monuments. The principal means of architectural expression slowly migrated from the structure to the décor, but the messages that were conveyed remained largely the same. Reliefs have been recovered from several fifth dynasty pharaonic funerary complexes, most notably from the pyramid complex of Sahure. The remains demonstrate that the artisans were developing more sophisticated sculptural skills in order to display the complex iconography.

One fine piece of relief work including zoomorphic and avian themes dates to the reign of the first pharaoh of the fifth dynasty, Userkaf (Figure 5-1). It was reused during the Middle Kingdom when the stone it was carved on was taken from Userkaf's complex to the pyramid of Amenemhat I at Lisht. It was found there in 1991 above a robber's tunnel, reused in that structure. ${ }^{125}$ The scene shows Horus overhead, "The Behdite, great god of multi-colored plumage who comes forth from the horizon", with the goddess Wadjet below-left, lady of the per-nu shrine of Lower Egypt, of $\mathrm{Pe} / \mathrm{Pt}$ (Buto). She faces Horus who stands on the pharaoh's name contained in a serekh and directly in front of his prenomen in a cartouche. The snake goddess presents the symbols of the $w 3 s$ scepter and the shen-ring to the beak of Horus, who wears the double crown. Nekhbet, the goddess of Upper Egypt, of Nekheb near Hierakonpolis, does the same on the lower right-side. The scene repeats and reinforces the core message that the living Horus is the one who has the power to encircle, unite, and give life to the two lands.


Fig. 5-2. Iconographic panels carved on granite columns at the pyramid of Sahure reflect the orientation and location of the monument (D. Lightbody).

A similar pair of panels (Figure 5-2) has survived on a pair of rose granite columns dating to the reign of Userkaf's successor, the pharaoh Sahure. They stood side-by-side in the courtyard outside his pyramid on its east side. The column to the north of the central courtyard shows the cartouche and serekh of Sahure, surmounted by Horus wearing red dšrt crown of northern Lower Egypt. He faces the Wadjet goddess of the Delta who holds forth a shen-ring and the w3s scepter emblems. ${ }^{126}$

On the south side of the courtyard was a second column adorned with a panel that is a mirror image of the first one, apart from the fact that the reversed iconography reflects the emblems of southern Upper Egypt. Horus wears the white $h \underline{d} d t$ crown of southern Upper Egypt, and faces Nekhbet, the goddess of the southern land and Nekheb near Heirakonpolis. This is a fine example of how iconography in these pharaonic cult monuments reflected the political and the geographical contexts in which they were constructed, and even the varied flora and fauna of the two lands. Only by understanding these contexts and by analyzing the iconography with respect to the monument's particular location can the scenes be properly interpreted.

The Palermo Stone (Figure 5-3) is a monumental panel on which were carved historical records thought to date from the first half of the fifth dynasty. The inscribed tables on the main Palermo

[^16]panel and on its associated fragments are important sources for reconstructing the Old Kingdom timeline and the texts are often referred to as the royal annals. Entries on the inscribed stone grid recorded major events for each year of the reigns of the pharaohs listed, who lived from the first through fifth dynasties. Most notably with respect to this study is the repeated mention of a ritual that can be translated as "unification of the two lands, going around the walls". The glyphs used can be transliterated as $s m 3 t 3 w y$ phr $h 3$ inbw. This ritual was recorded as being performed on the first year of the reigns of new pharaohs, as well as at other times during their reigns. ${ }^{127}$ According to Toby Wilkinson, the heb-sed ritual mirrored these rituals recorded on the Palermo Stone. The pharaoh performed such ritual runs around the walls of the palaces and towns of his territory, including of Memphis. The "circuit of the wall" was closely related to the main ritual of the sed-festival, ${ }^{128}$ and its emphasis was on the king's authority and territory, and on the encircling unification and protection of the two lands.

Fig. 5-3. Entry from the Palermo stone describing rituals of encirclement (D. Lightbody).


Fig. 5-4. Entry from the Abusir papyri describing a ritual circuit followed by the priests (D. Lightbody).

This close relationship between the pharaoh, state rituals, and the state's pharaonic architecture is represented in many other entries on the Palermo stone. Some reference the building of temples, and the laying out of ground plans using the "stretching of the cord" ritual. While the pharaoh's principal role was keeping Egypt united and protected through rituals of encircling magic, he was also expected to inaugurate new construction projects and create new cult places where more rituals could be performed. The self-preservation society of ancient Egypt was in full swing during the Old Kingdom.

Another set of documents called the Abusir Papyri, dating from the end of the fifth dynasty have been recovered. They give insights into the operation of the pharaonic cults in the Memphite necropolis. ${ }^{129}$ The papyri record details of the daily duties of the cult priests at the pyramid of Djedkare-Isesi in Saqqara (although the documents are referred to by the name of the location

[^17]they were ultimately found, which was within the pyramid complexes at Abusir). ${ }^{130}$
One ritual referenced several times on the papyri (Figure 5-4) is a circumambulation undertaken by the priests who would walk up through the temples and up to, and around, the pyramid. ${ }^{131}$ The duties carried out by these caretakers of the eternal cult included inspections of cult paraphernalia, checking the security of doors and walls, ${ }^{132}$ and leaving and collecting "reversion offerings", signified by a shen-ring symbol. ${ }^{133}$ These would have been left at a monumental $h \boldsymbol{h t p}$ offering table

Fig. 5-5. 5 Statue of Neferefre protected and encircled by Horus with shens (D. Lightbody).

in front of the east side of the pyramid. The cult meals were placed there so that the ritual content could be consumed by the pharaoh's $k^{c}$ soul. There may have been pairs of steles flanking that location representing Upper and Lower Egypt. The food would later have been collected by the priests for their own consumption, hence "reverting" to those who had offered it.

The papyri mention w3t hen ntr phrww $h \mathbf{h} \boldsymbol{h} r$ r: the "path of the god's servant (priest) going up to and around at the pyramid". ${ }^{134}$

As they circled through the mortuary temple on their daily rounds, and around the pyramid, the

[^18]priests may also have paid homage to fine statues of the interred pharaoh, like the rose limestone statue of the pharaoh Neferefre recovered from his mortuary temple at Abusir. ${ }^{135}$ This statue (Figure 5-5) is similar to the well-known green diorite statue of Khafre, but in this example Horus is shown holding the shen-rings to the sides of the pharaoh's neck, as if to emphasize that the pharaoh is not just protected by the wings of the falcon, but is encircled with magical protection.

It is useful to consider this iconographic connection between Horus and the shen-rings more closely. The association was probably not only due to the encircling path taken during the heb-sed ritual; it may have taken direct inspiration from the way the birds flew in the heavens above.

Visitors to Egyptian archaeological sites may note hawks and falcons gyring over the edge of the desert and above temples and tombs. The necropolises of the west bank are particularly useful for birds who utilize updrafts generated by rocky outcrops, and the thermals created as the stones warm under the sun. The birds hunt and travel up and down the Nile on these rising air currents, adopting a circular gyring circular motion as they go around. Vultures, one of the only other birds often shown holding shens, often form "kettles" of several birds who gyre together in helix-like formations. This efficient flight mode allows them to fly without flapping their wings.

The association between the pharaohs, the falcons, and the tombs built on the outcrops at the edge of the western desert may, therefore, have been inspired by this encircling avian behavior that the ancient Egyptians observed in their natural world. As will be shown in the following chapters, it seems that this gyring motion also served as an inspiration for the designs of fine bowls and jars decorated with falcons that were used in the pharaonic cult rituals of the fifth and sixth dynasties.

## A group of decorated vessels

Several examples of rare $h s$ and $n w$ jars, pots, and larger bowls recovered from fifth dynasty sites appear to have formed a coherent type possibly used in pharaonic mortuary cult rituals. The iconography on the vessels shares motifs directly associated with the reigning pharaoh, including high-status emblems such as falcons, shen-rings, serekhs, and cartouches, sometimes arranged within a "cosmic frame" device. ${ }^{136}$ A semiotic analysis of the graphical symbols on the vessels suggests that they relate to the rituals in which they were used. Concepts of protection, encirclement, vigilance, virility, and purification were linked to the natural and architectural environments in which the vessels were made and employed. Together, these constituted the coherent symbolic system that was integral to the pharaoh's mortuary cult, as well as the larger mechanisms of pharaonic rule.

Fine stone vessels were integral to elite funerary assemblages from the Early Dynastic Period onwards. Predynastic decorative scenes painted on ceramic vessels were gradually dropped in favor of finely-made but undecorated carved stone vessels. ${ }^{137}$ At the start of the Old Kingdom, hundreds such jars were placed in the subterranean chambers of the Step Pyramid of third dynasty pharaoh Djoser at Saqqara. By the fifth dynasty, a few elite stone vessels were once again decorated, but with the new repertoire of iconography directly associated with the Old Kingdom pharaonic regime and its cults. Most of the globular or spheroid jars described here have been studied previously, ${ }^{138}$

135 Discovered during excavations by Verner in 1984. Now in Cairo JE 98171. See M. Verner, Abusir: Realm of Osiris (Cairo: AUC, 2002), 112, 129.

136 Spieser, Les Noms Du Pharaon: Comme Etres Autonomes Au Nouvel Empire, 23; H. Frankfort, Kingship and the Gods (Chicago: University of Chicago Press, 1978), 38.
137 The Early Dynastic stone jars from Hierakonpolis discussed earlier in this study that were decorated with an iconographic arrangement including the sema-tawi motif may be related to this late Old Kingdom pharaonic vessel group.
138 C. Ziegler, "Sur quelques vases inscrits de l'Ancien Empire", in Études Sur l'Ancien Empire et la nécropole de Saqqâra dédiées à Jean-Philippe Lauer, eds. C. Berger and B. Mathieu (Montpellier: Université Paul Valéry, 1997).
but the study was extended by the current author in several respects. ${ }^{139}$ In addition to the globular vessels, the group can include a tall decorated $h s$ vase and an elaborate serving bowl shown in a relief scene in the mortuary temple of Pepi II. Although the morphologies of these additional vessels are substantially different, they carry iconography that is comparable and they were also recovered from, or represented in, pharaonic pyramid complexes.

Within the Old Kingdom context, the repertoire of symbols on these vessels should be considered not just elite, but pharaonic. If considered alongside iconography from the New Kingdom or Late Period the repertoire may seem unremarkable or even typical. Falcons, shen-rings, and feather (rishi) motifs were by that time widely used in non-pharaonic and even non-elite contexts. During the Old Kingdom, however, symbols such as Horus and the shen-rings did not appear in contexts outside of those directly associated with the pharaoh or his closest family members. These bowls and jars were clearly intended to be used by or for the pharaoh, and were most likely made by members of the court. ${ }^{140}$ It is clear how discrete this usage was as even the highest status nonpharaonic mastabas of the period did not display iconography from that restricted pharaonic group or schema. For that reason, the role of this group of vessels is considered with respect to Old Kingdom pharaonic material culture, rituals, and tombs.

The description of the bowls that follows is not intended to include full technical details or a complete contextualization of every aspect of these vessels. The focus is on the major graphical and morphological elements and the ways in which these were arranged together to create meaning.

Although avian symbolism was ubiquitous in the pharaonic iconography of the Old Kingdom, it was increasingly recycled and used in novel ways during the fifth dynasty. The iconography expressed important concepts directly linked to the symbolic protection of the pharaoh and his monuments. The graphical symbolism indicates that the vessels may have been used in circumambulation rituals integral to the pharaoh's mortuary cult, emphasizing the encircling protection and purification of his monumental tomb and the sustenance of his cult. These vessels are likely to have been manufactured for use in those rituals, and reflected the meanings expressed through them.

The first and oldest vessel included in the group is the most elaborate (Figure 5-6). It is a large, 45 cm tall, 18 cm diameter, fifth dynasty libation jar made of sycamore ${ }^{141}$ wood and covered with blue green faience inlays and gold leaf detailing. The vase, of a type known as $h s$ in ancient Egypt, ${ }^{142}$ was reconstructed by Ludwig Borchardt based on the pieces he recovered from the funerary temple of the pyramid of Neferirkare at Abusir. ${ }^{143}$ The reconstruction now stands alongside the pieces collected during the excavations. ${ }^{144}$ The decorated form of the jar represents a falcon's streamlined body including its individual feathers, folded wings, and slightly flared tail. A large eye of Horus or $w 3 d t$ eye sits upon the shoulder of the vase, close to where the raptor's head would be. The falcon being referenced was the pharaoh's own guardian; the god Horus.

Shen-rings alternate with symbols of the fertility god Min along a band around the waist of the jar.

[^19]The hieroglyphs above the band constitute a typical dedicatory formula based on the titulary of Neferirkare: "Life to Horus, strong of appearances, of the sedge and the bee, who has appeared by means of the two ladies, of golden powers, Neferirkare, given life, power, authority, and health forever". ${ }^{145}$ The theme of pharaonic virility invoked by the fertility god Min is perhaps deliberately echoing the title of the pharaoh used here, $W \operatorname{sr} r \underline{h} \cdot w$, "Horus, whose appearances are powerful" or "strong of appearances". It is thought that the original vessel was solid so its primary purpose was as a ceremonial or ornamental piece. Although not depicted on temple walls of the Old Kingdom, other examples of these $h s$ shaped vessels have been found in similar contexts, although badly fragmented.


Fig. 5-6. Line drawing of the decorated faconiform $h s$ vase from the reign of Neferirkare (D. Lightbody); followed by a photograph of the reconstructed $h s$ vase (A. Webber). The close-up shows the shen-rings on the recovered fragments positioned around the waist band (A. Webber).

The second vase in the group is a finely decorated spheroid travertine/Egyptian alabaster ${ }^{146}$ jar now kept in the Louvre (Figures 5-7 \& 5-8). It dates to the reign of the pharaoh Unas in the fifth dynasty, based on the cartouche included in the design, and the remainder of the iconography. ${ }^{147}$ The vase carries an encircling decorative arrangement depicting the falcon Horus holding shenrings. The symbolic encircling protection of the shens is carried around to the reverse of the jar by a pair of uraeii snakes extending out from the shens held in the falcon's claws, running round in either direction. On the opposite side of the bowl the heads of the snakes hold ankh symbols of life on either side of the horizontally oriented cartouche containing the prenomen of the pharaoh. The whole arrangement essentially emphasizes the underlying meaning of the cartouche as a device associated with Horus for encircling and protecting the life and name of the pharaoh. The nswt-

145 Ibid., 346. Translation is based on the recovered fragments.
146 There is a degree of confusion regarding the different names employed for this particular stone. It is often called calcite but also Egyptian alabaster, alabaster, or calcite-alabaster. It should probably be referred to as travertine (Egyptian alabaster). See P.T. Nicholson and I. Shaw, Ancient Egyptian Materials and Technology (Cambridge: Cambridge University Press, 2000 ), 59.

147 The vase is on display in the Louvre as E 32372. It is 17 cm tall and 13.2 cm in diameter.
bity title signifying the king of Upper and Lower Egypt is included above the horizontal cartouche.


Fig. 5-7. Line drawing showing the iconography on the Egyptian travertine bowl including Horus and shen-ring iconography. Dedicated to Unas and now in the Louvre (D. Lightbody).

A thread of continuity running from the Pre-Dynastic Period can be seen in the design of this globular travertine/Egyptian alabaster stone vessel. The white material chosen and the avian themes suggest that it is a skeuomorph of the ostrich egg shell vessels used in earlier periods. ${ }^{148}$ Ostrich eggs with ritual functions were sometimes found in Predynastic burials including at Hierakonpolis, and examples are also known from the Old Kingdom (see item 57 in the catalogue here). The choice of white travertine/Egyptian alabaster stone as the material for the globular stone jar supports an association with white ostrich eggs and with avian symbolism in general. ${ }^{149}$

A similar fifth dynasty decorated globular travertine/Egyptian alabaster jar from the reign of Unas is now kept in the Oriental Institute in Chicago. ${ }^{150}$ It is larger, at 26.2 cm tall ${ }^{151}$ with a rim diameter of 18.7 cm (Figures 5-9 \& 5-10). Its motifs incorporate two large falcons with spread wings holding

148 R.F. Friedman and A.H. Muir, "Analysis of Predynastic Ostrich Eggshells from Hierakonpolis and Beyond", in Egypt at Its Origins 3, eds. R.F. Friedman and P.N. Fiske (Leuven: Peters, 2011).
149 N. Cherpion, "L'oeuf D'autruche Du Mastaba III", in Le mastaba de Khentika, ed. G. Castel (Cairo: IFAO, 2001). Attesting to the pervasive nature of the avian theme is a 6th dynasty jar made from an ostrich egg shell found in the mastaba of Khentika in the Dakhla Oasis of the western desert, see catalogue entry 57 at the end of this publication. This is now in Cairo, EM JE 98774. A unique aspect of the jar is the positioning of a shen-ring around the circular opening at the top of the egg into which a stopper is placed. The iconography seems naïve when compared to the other vessels, and given its relatively isolated find spot and the use of a more traditional material it seems likely that this was an attempt to copy the pharaonic ritual vessels, rather than actually being a pharaonic ritual vessel. As such it may be one of the earliest examples of this type of pharaonic symbolism being used indiscreetly, outside a pharaonic context.
150 R. Bailleul-LeSuer, Between Heaven and Earth. Birds in Ancient Egypt (Chicago: Oriental Institute Museum Publications, 2012), 206. See this publication for a general discussion of these vessels by R. Shonkwiler, and the associated article R. Shonkwiler, "Sheltering Wings: Birds as Symbols of Protection in Ancient Egypt", in Between Heaven and Earth. Birds in Ancient Egypt, edited by R. Bailleul-LeSuer (Chicago: Oriental Institute Museum Publications, 2012). The Unas travertine/Egyptian alabaster vessel now in the collection of the Oriental Institute in Chicago and is catalogued as OIM 13947.
151 It should be noted that this is very close to half a cubit tall. It seems that the artisan had started working with a solid block of travertine/Egyptian alabaster of half a cubit in height. The block would probably have been cut to that size from the quarries in a way that meant the translucent band of rock was at the appropriate level in the stone for the intended design. It was most likely sourced from the Hatnub travertine quarry, which was the pre-eminent source of travertine/Egyptian alabaster from the pyramid age onwards. The bowl was not apparently wheel-carved.
shen-rings in their claws on opposing sides of the jar. A square "cosmic frame" panel between the falcons' wings on one side includes the cartouche and serekh of the pharaoh and a formulaic dedicatory inscription that emphasizes the pharaoh's name $w 3 d-t s . w(y)$, the "flourishing of the two lands". The pseudo-symmetrical iconography within this framed panel emphasizes the two lands theme, and includes the paired goddesses representing the rule of Upper and Lower Egypt. The pair of falcons encircling the bowl also echo this duality.

A unique feature of this bowl is a single shen-ring depicted in isolation between the wingtips of the paired falcons, on the opposite side from the square titulary panel. This emphasizes the principal theme of encirclement and protection. After close comparison of the details of the motifs on this vessel with the iconography on the jar from the Louvre, which is also dedicated to Unas, it seems likely that the same artisan was responsible for creating both vessels. The lines used to draw the falcon's head in particular are almost identical in form and number in each case. Another notable aspect of this bowl from Chicago is that both the falcons face to the right from the viewer's perspective, invoking a counter clockwise orientation in the iconography when viewed from above,


Fig. 5-8. Photograph of the Unas bowl in the Louvre. Note the light-colored encircling band, which is not a reflection but a natural characteristic of the stone incorporated into the design of the artefact (A. Snyder Payne).
and this proved to be the rule for all such similar Old Kingdom vessels.
Several aspects of this bowl in Chicago intrigued the author, so a special visit was made to the Oriental Institute to study it in detail. As well as the inclusion of an extra shen-ring in the design, the fact that the stone used to make it included a large and rather obvious stained band running through the main design seemed anomalous, particularly when compared to the finer stone used to make the comparable Louvre example. While the inscription was sophisticated, it seemed to be very lightly incised into the stone. The combined effect of the stained stone and the lightly chiseled design made it difficult to discern the motif clearly. The hieroglyphs, however, were of a quality and form that suggested they had been created by a scribe who was also a skilled artisan, so it seemed strange that such a piece of stone would have been used in such a high-status context.

The reason for this combination became clear when the bowl was examined more closely in Chicago. After studying the travertine/Egyptian alabaster, it was noted that it had been deliberately selected by the artisan because the block contained a band of translucent stone running through the entire block. When lit from behind or within, the stain revealed itself to be a red-orange colored band running around the waist of the bowl and right through the motifs (Figure 5-10). The
artisan apparently incorporated the naturally occurring translucent band within the iconography, which was placed directly over the translucent sections. ${ }^{152}$ The seemingly anomalous shen-ring was positioned right over the translucent band on the reverse side, as if the artisan had deliberately indicated the special character of the stone incorporated into the design. Once light is shone into or placed within or behind the bowl, the translucent band lights up brightly with a red-orange color that is immediately reminiscent of a solar disc and, therefore, deeply meaningful within the pharaonic context. ${ }^{153}$ The design's incision depth can also be explained by this observation, as the artisan was hesitant to cut deeper in case excess light was allowed through the material. Some semitranslucent areas in the stone away from the main band indicate that this was a real risk.


Fig. 5-9. Line drawing showing Horus and shen-ring iconography on the travertine bowl dedicated to Unas, now in Chicago Oriental Institute. The single shen-ring on the left view is anomalous. The panel on the right is arranged within a "cosmic frame" device (D. Lightbody).

Background research on the geology of travertine/Egyptian alabaster shows that this type of stone builds up in caverns and fissures over time, rather like stalactites and stalagmites. ${ }^{154}$ The 'flow stone' created is produced by limestone solutions that dry and precipitate as secondary mineral deposits in horizontal layers over long periods of time. The layers crystalize in various ways depending on the climatic conditions at the time they form, and certain layers develop distinctive crystals that can react with natural radiation in the rock to form translucent amber colored or opaque white bands. These are what the artisan had used so creatively.

When additional photos were obtained of the Louvre vessel, possibly made by the same artisan, it became clear that it too had naturally occurring bands of lighter stone, but in the form of two

152 Although the Oriental Institute bowl and the Louvre bowl were apparently never studied in this way before, the positioning of motifs and designs with respect to translucent bands on similar vases and bowls was noted by Ziegler, 463.
153 Questions remain regarding how such bowls might have been illuminated in practice. There was no soot or charring on the inner side or inner rim of the Oriental Institute bowl, but it is wide enough so that an oil lamp enclosed within a smaller travertine cup could have been hung inside it, and secured there with small chains. Other possibilities are that the jar was positioned under a particular opening in the ceiling that would have allowed a shaft of light to shine down into the bowl at certain times of the day and year. The bowl could have been positioned in front of a window, or a reflective material could have been used to direct a beam into the bowl.
154 Geologists have studied the processes that cause colored translucent veins and bands in travertine. Travertine/Egyptian alabaster is a type of "speleothem"; a typical secondary mineral deposit found in limestone caverns and fissures and formed by limestone solutions. The liquid solution may have been heated by geothermal activity on the Rift Valley fault line. Precipitation surfaces form in horizontal layers as the minerals are deposited. Sometimes called 'flowstone'. Some bands have different geophysical and optical properties, depending on the environment at the time they were formed. Translucent colored calcite bands and opaque white calcite bands form in a matrix. Coloring is due to natural radioactivity in the minerals that acts on particular calcite crystal forms to produce translucent colored calcite. This can be bleached out by sunlight, so there is a possibility that the Louvre bowl also once had translucent amber bands. The principal travertine/Egyptian alabaster quarry was at Hatnub near Amarna. Inscriptions in the quarry show it was worked during the Old Kingdom. See J.A. Harrell, M. Broekmans, and D.I. Godfrey-Smith, "The Origin, Destruction, and Restoration of Colour in Egyptian Travertine", Archaeometry 49, no. 3 (2007).
opaque white circles surrounding the main motifs on either side of the vessel. The jar would have been manufactured in a similar way to the Oriental Institute bowl, but once the raw stone block


Fig. 5-10. Research photograph taken during study of the Unas bowl at the Oriental Institute in July 2018. Note the translucent encircling band incorporated into the design and iconography of the artefact (B. Alm, with the permission of the Oriental Institute).
was cut it was stood on end so that the layers formed by the strata ran vertically and so formed vertical rings after the block was cut into a globular shape. Looking again at the front face of the vessel, what had previously appeared to be reflections produced during photography were in fact circles of opaque light stone. The circle on one side incorporated a central dot so that the motif resembles the solar sign for Re. ${ }^{155}$ The ring on the front is positioned around and symmetrically over the central motif of the falcon Horus. On the reverse side, a similar band encircles the cartouche, and at the top the natural veins in the stone disperse in a feathered effect that coincides with the tips of the falcon's wings. The heads of the snakes attached to the shens also coincide positionally with the sides of the encircling band. These uraeus snakes are associated with Re, so it seems that the artisan utilized the special character of the stone and integrated it within the iconography to emphasize the solar, encircling, symbolism of the piece (Figure 5-11).

Given these common design elements and the similar ways in which the material was used, ${ }^{156}$ it seems likely that the Oriental Institute bowl and the Louvre jar were made by the same artist.

Within the context of Old Kingdom ritual and iconography, these bowls can be described as

[^20]156 It could be considered that the material used to make both these bowls was flawed, however, from the point of view of the artisan these natural veins may in facts have been seen as strengths that could be utilized to enhance the iconography. A comparable example may be the Japanese tradition of Kintsugi, or 'gold joining', where repairs made to valuable but broken ceramics are carried out with such care that the end result is considered to be more valuable than the original piece.
sophisticated encircling magic in material form. Perhaps more effectively than any other artifacts, these bowls incorporate all of the key concepts of the pharaoh's iconographic schema; the

Bowl A. Horizontal translucent amber-colored band in double-falcon bowl. OIM13947.

"Translucent colored calcite bands and opaque white calcite bands"

Jar B. Vertical opaque white bands in single falcon jar. Louvre E 32372


Fig. 5-11. Comparison of the solar symbolism of the travertine veining on the two vessels dedicated to Unas. The bowl in the Oriental Institute is above, and the jar in the Louvre is below. The circle over the falcon on the Louvre jar lower left incorporates a central dot, so that the veining forms the shape of the sign for the solar god Re (D. Lightbody).
cartouche, the gyring falcons, encircling rituals represented by the uniting shens, and encircling solar discs. The vessels were designed to be used in particular monuments set within particular landscapes, and they reflect the pharaonic cult contexts in every way possible. ${ }^{157}$

## Sixth Dynasty

Vessels of the type described above continued to be produced in the sixth dynasty. ${ }^{158} \mathrm{~A}$ similar travertine/Egyptian alabaster jar decorated with an inscription dedicated to the first pharaoh of the sixth dynasty, Teti, was found at Tell-Edfu in Upper Egypt. ${ }^{159}$ It is not discussed in detail here. The final vessel in the study addressed here is a newly reconstructed decorated serving bowl, based on a representation shown on fragments of a sixth dynasty relief from the pyramid complex of Pepi II Neferkare at Saqqara (Figure 6-1). A pair of falcons were shown positioned in opposition around the rim of this large serving vessel as if holding the bowl on either side where handles would have been placed. It is shown filled with lotus lilies and other produce.


Fig. 6-1. Reconstructed decorated serving bowl from the reign of Pepi II Neferkare (D. Lightbody).

Although difficult to perceive at first, the fine serving vessel is quite well depicted on the fragments collected from the mortuary temple of the pyramid. ${ }^{160}$ Enough of the relief survives and enough of the details of the rim and sides of the vessel survive so that the author was able to artistically reconstruct its overall form. The bowl was symmetrical, so the surviving linear outlines of the form depicted on either side could be used to reconstruct the form of the opposing side, and vice versa. Similarly, the decorative iconography on the vessel consisted of a symmetrical design, which

158 Fragments of several other bowls have been found. See Bailleul-LeSuer, 206. Similar tavertine/Egyptian alabaster bowls with panels but no falcons have also been found, c.f. travertine/Egyptian alabaster jar inscribed with panel for Pepi I celebrating his first sed festival, mentioning his pyramid at Saqqara. Egyptian Museum Berlin ÄM 7715 shows a priest with a portable offering table of vessels supported by a shen sign, possibly signifying a reversion offering, and similar to a representation in a text in the mastaba of Hnw (see catalogue entry 54 in the present volume).
159 B. Bruyere, Tell-Edfou. Fouilles franco-polonaises, rapports, 1 (Cairo: IFAO, 1937), 35, pls. xvii, xxii. This travertine/Egyptian alabaster jar is decorated with a dedicatory inscription for the first pharaoh of the 6th dynasty, Teti. It was found at Tell-Edfou in Upper Egypt. A long text in a circular band around the opening of the vessel reads 'Living Horus, who satisfies the two lands, king of Upper and Lower Egypt, Teti, son of Re, given life, stability, strength, forever'. This text is typical of the period. A smaller panel containing the pharaoh's serekh and cartouche is similar to that on the vessel dedicated to Unas in Chicago, although simpler. It reads 'Horus, king of Upper and Lower Egypt, given life forever'. It is part of the collection of the Egyptian Museum in Cairo (JE 6689). Other graphical details on this vessel include the use of scepters? or reeds? to extend the encircling symbolism from the shen-rings around the base of the vase via ankhs attached directly to the shens. A lotus flower rosette on the base of the vessel emphasizes the life-giving symbolism of the iconography. See catalogue below entry 56 for details.
allowed a full reconstruction of the iconography and morphology to be attempted.
It is a unique example of such a vessel from this period and is clearly different to the other vessels discussed in this article, however, it carries a repertoire of iconography that is in harmony with the schema used on the other vessels.


Fig. 6-2. The Pyramid Texts (Pyramid of Unas) manifest ritual concepts in stone (S. Brabin).
One question raised by the depiction in the relief was whether the falcons each had two heads, i.e., one each hidden on the far side of the scene, or if they only had one head each that looked towards the viewers of the two-dimensional representation. Based on comparison with the surviving vessels, it seems likely that the relief depicted a real vessel that had falcons that both looked to the right, as is the case for the vessel from Chicago that also has a pair of falcons. The falcons on all of the other vessels also look to the right from the viewer's point of view, i.e., in a counterclockwise direction if looking from above. The two-dimensional depiction in the relief, therefore, probably employed artistic license to avoid showing one falcon with no visible head.

Like the other vessels, this bowl's design focused on Horus, the shens, the pharaoh's prenomen in a cartouche, and the phrase "living forever". The relief in which this vessel is depicted was designed and constructed within the highest status architectural context; within the pharaoh's own pyramid complex. That context supports the proposition that this iconography was directly linked to the pharaoh during that period, and could not be used more generally.

In later periods, the semantic range of contexts in which these signs were used spread out substantially, but during the Old Kingdom usage was fastidiously limited and discrete; reserved for the pharaoh or his closest family members.

Beyond those direct symbolic references to the pharaoh, and the decorative, aesthetic value of the designs, were there more profound meanings associated with the repeated avian and emblematic concepts? Did they reference the encircling rituals and motifs designed into the pharaonic mortuary
architecture; a tradition that already dated back hundreds of years by the time these vessels were created?

The most striking element of the designs are the spread wings of the falcons encircling the vessels. This was undoubtedly a sign of protection. ${ }^{161}$ Wings provided a protective shield, most elaborately expressed in the feather motifs of rishi coffins in later periods. The concept of encircling was also expressed by the wings. As well as forming a physical barrier, the all-encompassing wings surrounded and embraced the enclosed vessels, and this concept was often emphasized by the shen-rings grasped in the falcons' claws. The concept of vigilance is also invoked by the symbolism. The falcon flies around on those wings and observes the world below with its hunter's eyes. Texts from all periods talk of the avian gods hovering above in protection, and of the circular, gyring, motion of those protectors in the sky: "When he hears the cry of Isis and Nephthys his heart goes about for them" ${ }^{162}$


# šnn.t mr pn ḥwt-ntr tn 

This pyramid and temple are encircled

Fig. 6-3. Phrase from Pyramid Text 534 attesting to the ritual encircling of the pyramid and its temple.
The group of vessels examined here was surely integral to the rituals held within the mortuary architecture, and hence the iconography is intimately related to that architectural context as well as the wider ideological structures of pharaonic status and rule. Several of the vessels studied here were excavated from particular architectural spaces indicating that they were used in the rituals carried out within those sacred spaces. The iconography may even have been reserved for use only within those ritually enclosed, sacred zones. The iconography on the vessels is, therefore, intimately related to the rituals of protection and purification of the pharaoh's own mortuary cult. It was designed to help ensure his eternal protection and successful rebirth in the afterlife, and the daily offerings of water and foods sustained the pharaoh's ability to keep Egypt unified and protected in the afterlife. The encircling, overseeing, vigilance of the living pharaoh's own patron god, the falcon Horus, was invoked through the iconography.

Festivals of territorial unification, rituals of virility and regeneration, eternal cults celebrating the ruling dynasty, and daily encircling rituals to maintain the integrity of the pharaoh's monumental architecture, shared an interconnected and inter-related iconographic system that ultimately drew inspiration from the natural world and the heavens above.

## Pyramid Texts

The pharaoh Unas was the first to decorate the walls inside his burial chamber with texts, right at the end of the fifth dynasty. Pharaohs of the sixth dynasty followed his lead. As well as acting

[^21]to guide the pharaoh's soul to the afterlife, ${ }^{163}$ their aim was to surround the tomb with magical protection. ${ }^{164}$ The Pyramid Texts are a long succession of litanies, offering formulae, prayers, and magical spells, some of which were recited only on the day of the funeral. They constitute some of the very oldest religious text known to human-kind, and they most likely record concepts that had until then only been documented on papyrus or learned orally (Figure 6-2).

Perhaps because it was becoming increasingly difficult to maintain uninterrupted eternal cults and supplies of offerings at the time these texts were first carved, the Pyramid Texts should be seen as an attempt to ensure continuity in case successors were unable to maintain the pharaonic cults. Whatever the political situation above ground, the pharaoh could perpetuate his daily cycles of rebirth, carry out his celestial and chthonic rituals, and maintain his place with the gods thanks to the power of written language alone. This change of focus perhaps indicates that the pharaonic culture was becoming less ambitious in outlook, and even rather fearful of the future. A succession of inward-looking sovereigns developed more inward-looking texts.

Evidence of ritual encirclement appears several times within these texts. "For you are Horus, surrounded by the protection of his eye ${ }^{3165}$ is written on the east wall of the tomb chamber, above the entrance, in the pyramid of Unas. The phrase uses the word $\check{s} n^{166}$ in this case to describe the encircling protection.

Representations of rituals in art and architecture served to ensure their re-enactment for all time. ${ }^{167}$ The practice of enshrining and petrifying the rituals by depicting them in decorative relief scenes seems to have evolved into the practice of describing the rituals in Pyramid Texts carved into the walls of the tomb chambers.

Textual evidence of ritual encirclement specifically applied as an architectural motif appears at the entrance of the sixth dynasty pyramid of Pepi I Meryre at South Saqqara (Figure 6-3). The Pyramid Texts on the internal entrance passage walls include a complete spell PT $534 \$ 1277$ that refers to a ritual of encircling protection for the pyramid using the word shen or shenu. ${ }^{168}$ Faulkner referred to this text as: "a spell for the king's tomb". ${ }^{169}$ Allen and Der Manuelian call it a "spell for protection of the tomb", ${ }^{170}$ while Mercer stated that it was "for the protection of the pyramid enclosure". ${ }^{171}$ Based on these three sources and additional sources addressing the use of the word shen in texts, the relevant section of the text, spell, or prayer is as follows:

Entrance corridor, north section, east side, lines 46-49:
46 1277a jw.n.(j) wpww js ḥtp-gbb djw tm

[^22]47 1277b wdn mr pn ḥwt-ntr tn n ppj $n k 3 . f$
48 1277c šnn.t mr pn ḥwt-ntrr tn n ppj $n$ kJ.f
49 1277d w「b jrt harw tw
"A Geb offering that Atum grants. This pyramid and temple are established for Pepi and for his ka. This pyramid and temple are encircled (shenned) for Pepi and for his ka. Pure is this eye of Horus".

This is a very clear statement of the concept of ritual encirclement and its association with temple and pyramid architecture. This text has been translated several times, but only one publication addresses its specific ritual meanings and architectural context. ${ }^{172}$ Osing carried out the most focused study and concluded that the protective function of this text is clearly related to its position at the entrance to the pyramid. He noted that it should ideally be located on the exterior casing stones of the pyramid's face, but that the scribes were probably wise to place it in the entrance corridor rather than on the exterior (thus ensuring its survival), and that its location there is appropriate. He also noted that the phrase "eye of Horus" is a metaphor for the whole encircled and purified pyramid complex. ${ }^{173}$ Hays described the text as concerned with the perpetuation of the cult, the longevity of the tomb complex, and the transmission of offerings from the gods to the deceased. ${ }^{174} \mathrm{He}$ noted that the reciter in the text is not the beneficiary, and as such it would have been a sacerdotal rite recited by an officiant who addressed the beneficiary directly, and also spoke about him. ${ }^{175}$ Hays described this type of prayer or spell as a sacerdotal motif particular to priestly recitations, enacted by the priests for the pharaoh.


Fig. 6-4. A line from Teti's Pyramid Texts describing encircling ritual concepts associated with the heavens above (D. Lightbody).

What Osing and Hays did not fully develop in their discussions, however, is what the expression "to encircle" entailed and what concepts it was associated with during the Old Kingdom. Osing understandably translated "shen" as one word, but it is argued here that it was associated with a whole schema of inter-related concepts in the ancient Egyptian mind. These encompassed circumambulation rituals, pharaonic architectural traditions, and the institutions and cults of kingship. ${ }^{176}$

The most difficult line of the text to understand is the first of the four lines: "A Geb offering that Atum grants". Avian contexts were common at the time. Geb was often associated with a goose during the Old Kingdom, as well as with the earth, and in light of the fact that the creator god Atum was thought to have been born from a cosmic egg on the primeval mound, this line perhaps constitutes an avian metaphor. The whole encircled pyramid complex is perhaps being compared

[^23]with a giant white goose egg being laid from the heavens above. ${ }^{177}$ This interpretation remains tentative, but it is worth recalling the frequent inclusion of eggs and egg-shaped vessels within burials dating back to Predynastic times.

De Trafford, who studied the architecture of the Pyramid Texts and the associated palace-façade decoration within the tomb chambers, noted that the texts cover the entire wall surface within the funerary chamber and extend in a continuous circuit around the space. ${ }^{178}$ She made a direct association between the iconography of the palace-façade walls in the tomb of Unas and the palace façade walls of the pyramid of Djoser, which is close to where the pyramid of Unas was located. ${ }^{179}$ Unas internalized, compressed, and "textualized" the encircling enclosure walls, attesting to the fact that monumental architecture was by that time effectively being replaced by texts. Unas merged the petrified oral dimension of the rituals with the enclosure walls themselves to create the iconic texts.

One further line from the Pyramid Texts (Figure 6-4) in the pyramid of Teti (274, §406c) makes it clear that the pharaoh was expected to travel around the skies in the afterlife, in a way that recalls the heb-sed rituals still being represented in the funerary temples outside these sixth dynasty pyramids:
iw dbn.n tti pt.wy tm.ty.wy phr.n.f idb.wy
"Teti has gone around the two skies, he has circumambulated the two banks". ${ }^{180}$
The two banks referred to here are usually associated with the banks and extents of the Nile, and should perhaps be associated with celestial Nile in this context. Now known as the Milky Way, this heavenly Nile was the band of stars which stretches across the night sky from north to south.

Egyptologist Toby Wilkinson remarks that the heb-sed ritual fulfilled much the same function as the circuit of the wall performed at Memphis by the pharaoh on his coronation day. ${ }^{181}$ Within the funerary temple context, the presence of the heb-sed motif implies that the pharaoh was expected to continue to perform these rituals and to act as the force that kept Egypt stable and unified, in the afterlife. This phrase from the Pyramid Texts alludes to the same conceptual framework.

The ritual concept of encirclement is expressed textually in this sentence from the Pyramid Texts, and it perhaps throws more light on the most enigmatic architectural features of Khufu's pyramid; the star/air shafts. The pharaoh's primary ritual function was to unify and lay claim to the two regions of Egypt, to create one stable territory running from the southern border to the Mediterranean Sea, both in the celestial realm above and on the earth below. It is, therefore, notable that the star/ air shafts leading out from the pharaoh Khufu's burial chamber aim towards the two skies, due north and due south. Although the ancient Egyptians left no written explanation for the purpose of the shafts in the Great Pyramid, the architectural, ritual, iconographic, and textual evidence outlined here indicates that the shafts were connected to this ideology of territorial encirclement and unification.

[^24]
## Later kingdoms

This section provides a general summary of significant examples of encircling symbolism applied in later phases of ancient Egyptian history, and in later examples of pharaonic tomb architecture. It is not intended to be a comprehensive survey, and is included primarily to contextualize the more detailed discussions above relating to the Old Kingdom.

The artifact from the later kingdoms that is most closely related to Old Kingdom rituals is a unique but fragmentary funerary liturgy on papyrus recovered from a 13th dynasty tomb beneath the Ramesseum. It is thought to contain information derived from or dating to the Old Kingdom era, perhaps from as early as the third dynasty, and it records a circumambulation ritual to be carried out on the day of the burial of the deceased. ${ }^{182}$ This is one of the few textual sources that refer to such a practice directly. Gardiner was keen to emphasize the importance of this fragmentary papyrus, given that its contents may be derived from the Old Kingdom. The papyrus describes how an entourage of commoners, nobles, and the pharaoh's children should perform four circumambulations of the tomb, sometimes walking, processing, or dancing in opposing directions, while mourning for the deceased on the way.


Fig. 7-1. Middle Kingdom coffin from the NY Metropolitan Museum incorporating shen-ring iconography (D. Lightbody).

Circumambulation rituals remained at the heart of the pharaonic belief and ceremonial system, and not just in the Memphite Necropolis. An important holiday at Abydos was the phr, Poker festival, literally meaning "the festival of going around", held in the vicinity of the Early Dynastic royal tombs. On the day of the prt ${ }^{〔} 3 t$ Great Procession the celebrants followed a circuitous path out into the desert and around the tomb of Djer that was thought to be the burial place of Osiris himself.

When they got there, according to a $12^{\text {th }}$ dynasty stele from Abydos, they held a $\operatorname{ddryt}$ tn $h r$-šn : vigil of Horus $\check{s} n .{ }^{183}$ Contrary to earlier attempts to translate the second part of this term as "Horus the fighter", it seems more likely that it referred to the concept of the encircling Horus; the protecting, vigilant, gyring falcon, the patron god of the ancient rulers. The festival appears to have been a ritual of renewal of the pharaoh's powers, when a new layer of gold leaf was applied to the cult statue. On the morning before departure, and on the morning after the vigil out in the desert, the $h 3 j$.k.r.j Haker ritual was performed, meaning "come down to me". ${ }^{184}$ It seems reasonable to propose that this rite called on the spirit of the ancestral falcon, to descend from the heavens into the cult statue and by extension into the living pharaoh.


Fig. 7-2. The cartouche shaped sarcophagus of Tuthmosis III, one of several examples from the 18th and 19th dynasties (D. Lightbody).

Encircling rituals were not only associated with tombs. As mentioned previously with respect to the Old Kingdom, accession rituals for new pharaohs included a rite known as the "circuit of the walls". ${ }^{185}$ After receiving the royal regalia, the new pharaoh was declared to be the god Horus and embarked on a circuit of the walls of Memphis in a ritual described as the "union of the two lands, circuit of the white walls." ${ }^{186}$ Given the protective aspect of these ritual tours of the walls, it seems reasonable to consider if the magical practice may have grown out of a practical task such as periodic inspections of walls carried out in order to ensure the integrity of the stone or mudbrick. It seems possible that the circumambulation routes of later Sokar festival processions around the outside of temple walls may have incorporated a ritualized practical component. ${ }^{187}$

[^25]The use of Old Kingdom pharaonic symbolism of this type seems to have become more limited during the Middle Kingdom, when the pharaohs were perhaps more concerned with taking practical measures to consolidate power rather than symbolic or magical ones. The beautiful pectorals of Queen Mereret and Princess Sithathoryunet should. however, be mentioned here. During the later Middle Kingdom the shen began to appear in contexts that were not pharaonic (Figure 7-1). Later Middle Kingdom coffins include examples where shen-rings and the palace façade/false door motif appear as a group on the coffin sides along with pairs of falcon eyes. ${ }^{188}$


Fig. 7-3. Scene showing Nephthys with a shen-ring at the foot of the cartouche shaped sarcophagus of Hatshepsut, recut for her father Tuthmosis I (D. Lightbody).

The symbol did, nevertheless, continue to be closely associated with pharaonic burials. In the New Kingdom, a whole series of 18th and 19th dynasty royal sarcophagi were manufactured in the form of cartouches (Figure 7-2). ${ }^{189}$ Many of the sarcophagi were made of red granite or were made to resemble red granite, and many were decorated with designs of the goddesses Isis and Nephthys holding shen-rings at the head and foot of the sarcophagi, respectively (Figure 7-3). The sarcophagus of Tuthmosis $I^{190}$ originally made for Hatshepsut (Figure 7-3), as well as those of Tuthmosis III (KV34), Senenmut (TT71), Amenhotep II (KV35), Merneptah (KV8), and the massive example made for Ramses III, ${ }^{191}$ were all cartouche shaped or were decorated with large

[^26]cartouches on the lids, and were either carved from red granite or painted to resemble the sacred stone from Aswan. Anthropoid coffins such as those of Tutankhamun were also decorated with rishi (feather) designs. The protective wings of Horus, or in some cases Nekhbet, ${ }^{192}$ sometimes enveloped these coffins entirely, but the claws of the avian deities still held prominent shen-rings over the chest or abdomen of the deceased, thus maintaining the traditional sign of protective encircling symbolism. This symbolism was later employed in non-royal contexts, such as on the $22^{\text {nd }}$ dynasty cartonnage mummy case of lady Tabes in Boston (Figure 7-4). ${ }^{193}$


Fig. 7-4. Cartonnage mummy casing with rishi decoration belonging to lady Tabes of the 22nd dynasty (D. Lightbody).

Pharaonic tomb chambers were sometimes cartouche shaped, such as the main burial chamber of the tomb of Tuthmosis III in the Valley of the Kings, KV34. ${ }^{194}$ That tomb synthesized several traditional symbols of protection and encirclement in an effective and quite novel manner. As well as being cartouche shaped and containing a cartouche shaped sarcophagus (Figure 7-2), the Amduat liturgy is depicted on the walls. Hour 5 includes a representation of the tomb of Osiris/Sokar shown in ovoid/ cartouche form, and rendered as if to resemble red granite. The shape used in the depiction has been associated with the ovoid form of the real tomb. It is shown under a pyramidal form that may reference the peak of el-Qurn far overhead, above the Valley of the Kings. ${ }^{195}$ At Giza, the pyramids were integral to the necropolis landscape, whereas for KV-34, the architects attempted to use the mountainous landscape above the hidden valley of Thebes as a proxy for the pyramid superstructure.

Given the importance that Akhenaten attached to the form of the circle, which to him was the

[^27]manifestation of the Aten, it is notable that the tombs of the Amarna Period placed significant iconographic emphasis on the cartouche form. The hieroglyphs in the tomb texts of the period often include a phrase that appears in the Great Hymn to the Aten: $n b$ šnn.t, followed by a cartouche determinative. This is typically translated as "lord of all that the Aten encircles".


Fig. 7-5. Cartouche-shaped anthropoid sarcophagus originally belonging to the late 19th dynasty ruler Queen Tausret/Tawosret. Recarved for the 20th dynasty Prince Amenherkhepeshef. The lid is surrounded by a mehen snake for unending protection (K. Gingell).

The royal sarcophagus of Amenherkhepshef, a son of Ramses III and the crown prince, is one of the most abundantly decorated examples of the schema of iconography covered here. It was made in the $20^{\text {th }}$ dynasty and was later found under rubble in KV13 at the head of the Valley of the Kings. ${ }^{196}$ The red granite sarcophagus is simultaneously anthropoid and ovoid and is heavily decorated with protective encircling motifs (Figure 7-5). The hard stone is roughly cut, but the symbolism can be studied once it has been traced out diagrammatically. The iconography incorporates an encircling 'Mehen' snake, a protective deity who coils around the sun god Re in his journey through the night. Flanking this Mehen snake are ram headed vultures holding shen-rings, and two large wadjet eyes of Horus. Goddesses spread wings around more shen-rings, and guard the top of the sarcophagus, while Isis and Nephthys hold shen-rings against the encircling snake running around the foot of the sarcophagus. ${ }^{197}$

The shen symbol remained closely associated with royal tomb designs, but it did not remain restricted to that context during the New Kingdom. The $18^{\text {th }}$ dynasty vizier Rekhmire was one of the first to incorporate the shen-ring symbol in his tomb, TT100, where it was placed above the

196 The sarcophagus from KV 13, the tomb of the late 19th dynasty chancellor Bay, originally belonged to the late 19th dynasty ruler Queen Tausret/Tawosret. It was recarved in the area of the wig to show the youth sidelock for the 20th dynasty prince Amenherkhepeshef. The remains of the vulture crown from the original can be seen, and most of the incised decoration except for the names and title of Amenherkhepeshef are original. The name of Tausret appears in a cartouche on the head end of the lid. The lid has been re-mounted on the sarcophagus box. See also several articles by Hartwig Altenmueller who worked in KV 13, and N. Reeves and R.H. Wilkinson, The Complete Valley of the Kings (London: Thames and Hudson, 1996), 154.
197 The mehen serpent also appears on the granite sarcophagus lid of Sety II in KV 15, although the head end is missing. Another example of the mehen snake appears on the vertical sides of the lid of Ramesses III's granite sarcophagus, now in the Fitzwilliam Museum, Cambridge. The mehen snake appears on the first and second lids of the Merenptah granite sarcophagi; the second lid is also cartouche shaped. Note that on the third granite lid of the Merenptah sarcophagus, re-used for the burial of Psusennes I in Tanis, there is a slightly raised cartouche-shaped area with a rounded top and flat bottom, surrounding the Osiride effigy, although no mehen snake is shown. It seems reasonable to see a relationship between the cartouche/ extended shen and the mehen snake. This serpent also appears in the 7 th hour of the Amduat upper register surmounting (and protecting) the seated figure of Osiris. Together, these indicate an ongoing tradition of surrounding ovoid symbolism as a protective motif.
central panel of his false-door between a pair of falcon eyes. ${ }^{198}$ Several of the tombs of the 19th dynasty artisans from Deir el-Medina who built the royal tombs of the Valley of the Kings were also decorated with prominent shen-rings, such as TT1 which belonged to Sennedjem (Figure 7-6). A shen-ring was painted in a central position at the top of the end wall of his tomb and this arrangement was found in several New Kingdom tombs. As the tomb had a barrel vaulted ceiling, the shen was drawn high in the arched area, at the top of the end wall, where it was flanked by a pair of wadjet-falcon eyes providing vigilant protection around the whole tomb. By the later New Kingdom this arrangement was typically found on funerary steles belonging to people from all walks of life. The symbols were often incorporated into the lunette; the arched form at the top of the stele slabs, reflecting its position in the vaulted tombs. The motif was used extensively on steles from that time on.


Fig. 7-6. Scene from the arched end wall of the 19th dynasty tomb of Sennedjem at Deir el-Medina (D. Lightbody).

These spreading contexts masked the older meanings associated with the pharaonic architecture, but by tracking the diachronic developments back in time, and by separating out the older material and studying it independently, the older archaeological contexts can be reconstructed and understood.


Fig. 7-7. Arrangement from the 18th dynasty stele of Montuher (F. Monnier).

One final artifact of relevance from the New Kingdom is the earliest known depiction of pyramids and the sphinx. The scene is on the $18^{\text {th }}$ dynasty stele of the scribe Montuher, found by Selim Hassan during his excavations around the sphinx in the 1930s. ${ }^{199}$ The falcon Horus carrying the shen-ring is depicted, flying beside the Great Pyramid and over the great sphinx in its manifestation as Hor-em-akhet; Horus in the Horizon (Figure 7-7). This may indicate that the associations between Horus, the shen-ring, and the pyramids of Giza were still remembered in certain quarters at that time.

[^28]
## Conclusions

When taken together, the information set out in this study constitutes a powerful case demonstrating that encircling symbolism was an important ritual concept expressed in ancient Egyptian cults, artworks, and pharaonic architecture. ${ }^{200}$ If the sequence of events described above accurately reflects historical reality, then it seems that the conclusions drawn by Flinders Petrie at the end of the $19^{\text {th }}$ century were correct. Based on the evidence he found, he concluded that we should grant that these encircling architectural motifs were in the builders' original designs. The current study supports that conclusion, and the intention here was to provide additional evidence and interpretation to reinforce Petrie's conclusions.

The research process integrated different classes of evidence and the analysis of that evidence was presented in one continuous historical, meaningful, narrative. A logical sequence of concepts that developed over time was traced out and an attempt was made to uncover and understand the thought processes followed by the ancient Egyptians themselves. By applying this contextualized inter-disciplinary approach, it became clear that the ground plans, iconography, texts, and decoration can be best understood when viewed from the perspective of the ancient culture and its ritual environment, which belonged to a very particular geographical region.

It is difficult to develop historical understanding from descriptions of individual artifacts alone, no matter how detailed those descriptions are. A continuous narrative, however, drawing on information from a number of different sources, within which artefacts or monuments are addressed sequentially and placed on a timeline of ancient history, can reveal the trajectory of meanings present in the ancient Egyptian mind. The ancient Egyptians schematized their world in a way that could be communicated and learned through ritual, iconography, and architecture. The structural integrity of that schema means that it can now be reconstructed as it was understood by members of that ancient culture. We can move beyond description, towards interpretation, and arrive at a meaningful understanding of the recovered archaeological material, and surveyed data.

The architectural and iconographic analysis in this study throws light on a number of related issues, including the early development of mathematical concepts and the development of the cartouche symbol. The emergence and adoption of the cartouche as the principal sign of the pharaoh makes good sense when understood with respect to the rituals performed by the pharaoh and the associated political and architectural developments.

In order to differentiate the pharaoh from his peers and elevate his status, a new form of tomb architecture and a closely related new symbol were adopted. Both of these strategies emphasized the importance of encircling and protecting the pharaoh, in life and in the afterlife.

The pharaohs and the priests of ancient Egypt performed rituals evoking the encircling protection of the falcon Horus, in the courtyards of their temples and tombs and around the walls of their towns. Iconography depicting those rituals adorned the walls of the pharaonic monuments, and the associated concepts were recorded in texts that have come down to us. The symbolism was manifested in the principal exterior dimensions of the monumental pharaonic architecture, and at the heart of their eternal resting places. The architecture of the Horus king's tomb was ultimately inspired by real falcons and hawks, circling high over the Memphite necropolis.

## Glossary

Canon: The body of rules, principles, or standards accepted as axiomatic in a field of study or art. Recognized, authoritative, conventions used in an artistic style or movement.

Cartouche: The encircling graphical symbol that surrounded the pharaoh's chosen prenomen. The name had special relevance to the pharaoh and was chosen at the start of the reign. It constituted one part of the pharaoh's extended titulary. Graphically, it was an extended version of the shenring, and was represented as two loops of rope tied into a flattened knot at the bottom. First used in the late third dynasty. It was used to refer to the pharaoh even after death.

Lunette: An area framed by an arch or vault.
Schema: An underlying organizational pattern or structure; a conceptual framework that organized component parts under one structure. A small number of schematic elements usually characterized ancient cultures.

Seked: (or seqed) is an ancient Egyptian term describing the inclination of the faces of a pyramid. The system was based on the Egyptian's length measure known as the cubit. The cubit was subdivided into seven palms, each of which was sub-divided into four digits. The inclination of measured slopes was expressed as the number of palms and digits displaced horizontally for each cubit rise.

Sema-tawi: A compound symbol representing the phrase 'unite the two lands', made up of three basic parts. On either side are the emblematic plants of Upper and Lower Egypt; the lily (Egyptian lotus), and the papyrus reed. They are shown tied together around a central symbol that is thought to represent a heart and lungs with its windpipe attached. This central element forms the axis of the motif and may represent the pharaoh as the heart that unites all Egypt, and the Nile river, which is the channel that brings life to all Egypt in the form of fresh water and fertile silt.

Serekh: A type of heraldic crest used in ancient Egypt. Like the cartouche, which developed later, it contained the pharaoh's name. Its distinctive rectilinear graphical form combined a view of a niched palace façade, or perhaps a false door, and the plan of a courtyard where the Horus name of the pharaoh was written in hieroglyphs. The word derives from the Egyptian word for façade, srh. It was in use as early as the Naqada II period, but without including hieroglyphs. Used only during the pharaoh's lifetime. Often surmounted by the Horus falcon.

Stela: An upright stone slab or pillar bearing an inscription or design and serving as a monument, tombstone, marker, or similar. The plural of stele used in this work is steles.

Temenos: A sacred area defined by a boundary; usually a wall. The word temenos is derived from the Greek té $\mu \varepsilon v o c ̧$. It is an area of land cut off and designated as an official domain, or a piece of land marked off from common use and dedicated to a god or gods, such as a sanctuary, holy grove, or holy precinct.

## Catalogue of shen iconography

This catalogue presents a comprehensive list of every known example of the shen-ring used in the iconography, material culture, and texts that have survived from the Old Kingdom and the Early Dynastic Periods, and by default, all preceding periods of ancient Egyptian history. Pyramid Texts that use the term $\check{s} n w$ rather than a representation of a shen-ring are not included here. Figure numbers included in the "References" column refer to artifacts illustrated in the current study, such as (Figure 1-2) at the end of the first row below. Readers are encouraged to contact the author if they are aware of any additional items that should be included in future versions of this catalogue.
\(\left.$$
\begin{array}{|l|l|l|l|l|}\hline \text { Dyn } & \text { Item. } & \text { Artefact description. } & \begin{array}{l}\text { Location \& } \\
\text { catalogue. }\end{array} & \text { References. } \\
\hline 1 & 1 & \begin{array}{l}\text { Ivory tag or box lid found in the tomb of pharaoh } \\
\text { Den in Abydos by Petrie. Earliest known shen-ring } \\
\text { depicted in Egyptian iconography. Serekh with hi- } \\
\text { eroglyphs of Den's name, surmounted by Horus. } \\
\text { Also, a uraeus snake and hieroglyph for gold above } \\
\text { the shen-ring. }\end{array} & \begin{array}{l}\text { British } \\
\text { Museum }\end{array} & \begin{array}{l}\text { (Figure 1-2); Petrie, } \\
\text { W.M.F., Royal Tombs of } \\
\text { the Earliest Dynasties II }\end{array} \\
\hline 2 & 2 & \begin{array}{l}\text { Calcite Jar with name of pharaoh Khasekhemwy. } \\
\text { Found in Hierakonpolis temple main deposit. Gift } \\
\text { from the Egyptian Research Account, excavated by } \\
\text { Petrie and Quibell. The group includes the vulture } \\
\text { goddess Nekhbet standing before the Horus king's no. } \\
\text { name, grasping the sema-tawi sign symbolizing the } \\
\text { unification of the two lands into one, while she } \\
\text { crushes a symbol representing the abbreviated } \\
\text { word for rebels, within the shen-ring. In this early } \\
\text { context the shen probably represents the encircle- } \\
\text { ment and captivity of the rebels rather than their } \\
\text { encircled protection. The fact they are within the } \\
\text { shen could represent an internal rebellion. The } \\
\text { same scene is repeated on items 3, 4, and 5. }\end{array} & \begin{array}{l}\text { Penn Mu- } \\
\text { seum }\end{array} & \begin{array}{l}\text { (Figure 2-1); Petrie, } \\
\text { W.M.F. and Quibell, J.E., } \\
\text { Hierakonpolis I (1900), } \\
\text { 11, pls. 36, 37, 38. Also } \\
\text { Silverman, D.P., (ed.) }\end{array}
$$ <br>

\hline Searching for Ancient\end{array}\right\}\)| Egypt (1997), 94. |
| :--- |


| 3 | 7 | Subterranean relief in situ under Step Pyramid of Djoser at Saqqara. Southernmost of three running north south under pyramid. Pharaoh runs ritual sed festival course protected by Horus with shen above. Text on left reads "at the southwest corner of the broadcourt of the sed festival court". Pair of half sky glyphs with shens on right associated with sed unification ritual. Wepwawet the Jackal standard above left. | In situ. | (Figures 3-2 through 3-4); Friedman, F. D. and Friedman, F., "The Underground Relief Panels of King Djoser at the Step Pyramid Complex", JARCE 32 (1995), 3. |
| :---: | :---: | :---: | :---: | :---: |
| 3 | 8 | Subterranean relief in situ under Step Pyramid of Djoser at Saqqara. Middle panel of three under pyramid. Pharaoh shown running the sed ritual between dnbw markers representing the limits of the land. Holds mks document which describes all that is "in the house" in his territory. Text on left reads "the white shrine of the great ones". Meaning unclear; ancestors? Wepwawet the jackal standard above left. | In situ. | (Figures 3-2 through 3-4); Friedman, F. D. and Friedman, F., "The Underground Relief Panels of King Djoser at the Step Pyramid Complex", JARCE 32 (1995), 3. |
| 3 | 9 | Subterranean relief in situ under Step Pyramid of Djoser at Saqqara Northernmost of three running north south under pyramid. Text on left reads "Standing in the shrine of Horus the Behdite". The Behdite is the falcon protective god and representative of Upper Egypt. Wepwawet the Jackal standard above left. Meaning of Scorpion remains unclear. Pair of half sky glyphs with shens on right associated with sed unification ritual. | In situ. | (Figures 3-2 through 3-4); Friedman, F. D. and Friedman, F., "The Underground Relief Panels of King Djoser at the Step Pyramid Complex", JARCE 32 (1995), 3. |
| 3 | 10 | Subterranean relief in situ under south tomb of Step Pyramid complex of Djoser at Saqqara. Southernmost of three running north south under southern tomb. Text on left reads "standing in the pr-wr shrine", the shrine of Upper Egypt. Wepwawet on standard above left, Horus with ankh above. Pair of half sky glyphs with shens on right associated with sed unification ritual. | In situ. | (Figures 3-2 through 3-4); Friedman, F. D. and Friedman, F., "The Underground Relief Panels of King Djoser at the Step Pyramid Complex", JARCE 32 (1995), 3. |
| 3 | 11 | Subterranean relief in situ under south tomb of Step Pyramid complex of Djoser at Saqqara. Middle of three under south tomb. Text on left reads "standing at the shrine of Horus of Letopolis"; a Lower Egyptian town. | In situ. | (Figures 3-2 through 3-4); Friedman, F. D. and Friedman, F., "The Underground Relief Panels of King Djoser at the Step Pyramid Complex", JARCE 32 (1995), 3. |
| 3 | 12 | Subterranean relief in situ under south tomb of Step Pyramid complex of Djoser at Saqqara. Northernmost of three running north south under south tomb. Text on left may read "creation" or "dedication of the sed festival statue at the southwest corner of the broad court". | In situ. | (Figures 3-2 through 3-4); Friedman, F. D. and Friedman, F., "The Underground Relief Panels of King Djoser at the Step Pyramid Complex", JARCE 32 (1995), 3. |


| 3 | 13 | Statue of stolist priest called Ankh in grey porphyroid granite with large shen encircling his neck. Seated with clasped hands. Height 62.5 cm. Inscription on statue reads "Stolist (priest) of Horus, overseer/fashioner of the 3 ms -scepter, Ankh". Reign of Djoser. Unknown provenance. It is notable that Horus is closely associated with the shen-ring during the period that this statue was made. Another statue (Rijksmuseum van Oudheden, Leiden AST 18, D93) probably of the same individual refers to him as an official of the city of Nekhen, i.e., Hierakonpolis. He appears to have held a special office or been nominated to perform a special ritual function, perhaps associated with Horus. | Louvre Museum <br> N40. | (Figure 3-5); See Allen, J.P. et al. (eds.), Egyptian Art in the Age of the Pyramids, Catalogue of Dynasty III, 184 \& 185. <br> Also Eaton-Krauss, M., "Two Masterpieces of Early Egyptian Statuary", OMRO 77 (1997). |
| :---: | :---: | :---: | :---: | :---: |
| 4 | 14 | Fragmented section of relief from the pillars of the Valley Temple of the Bent Pyramid of Snefru at Dahshur. This shows glyphs associated with the left side of a heb-sed scene including half-sky glyphs with shen-rings, and the bollards, and the bull tail near the heel of the running pharaoh. |  | Fakhry, A., The Monuments of Snefru at Dahshur. Volume 2. Part 1. The Temple Reliefs (1961), 66, fig 43. |
| 4 | 15 | Fragmented section of relief from the pillars of the Valley Temple of the Bent Pyramid of Snefru at Dahshur. This also shows glyphs associated with the left side of a heb-sed scene including half-sky glyphs with shen-rings, and the bollards, and the bull tail near the heel of the running pharaoh. |  | Fakhry, A., The Monuments of Snefru at Dahshur. Volume 2. Part 1. The Temple Reliefs (1961), 79, fig. 46. |
| 4 | 16 | Fragmented section of relief from the pillars of the Valley Temple of the Bent Pyramid of Snefru at Dahshur. This shows glyphs associated with the right side of a heb-sed scene including half-sky glyphs with shen-rings. |  | Fakhry, A., The Monuments of Snefru at Dahshur. Volume 2. Part 1. The Temple Reliefs (1961), 78, fig. 58. |
| 4 | 17 | Fragments of a relief from the pillars of the Valley Temple of the Bent Pyramid of Snefru at Dahshur. This shows glyphs associated with the right side of a heb-sed scene including half-sky glyphs with shen-rings. |  | Fakhry, A., The Monuments of Snefru at Dahshur. Volume 2. Part 1. The Temple Reliefs (1961), 79, figs. 59, 60 (right), 61. |
| 4 | 18 | Fragments of a relief from the pillars of the Valley Temple of the Bent Pyramid of Snefru at Dahshur. This shows glyphs associated with the right side of a heb-sed scene including half-sky glyphs with shen-rings. |  | Fakhry, A., The Monuments of Snefru at Dahshur. Volume 2. Part 1. The Temple Reliefs (1961), 86, fig. 68. |
| 4 | 19 | Fragments of a relief from the pillars of the Valley Temple of the Bent Pyramid of Snefru at Dahshur. This shows glyphs associated with the right side of a heb-sed scene including half-sky glyphs with shen-rings, also part of a flying Horus falcon carrying a shen-ring in its claws. |  | Fakhry, A., The Monuments of Snefru at Dahshur. Volume 2. Part 1. The Temple Reliefs (1961), 87, figs. 69, 70 |
| 4 | 20 | Fragments of a relief from the Valley Temple of the Bent Pyramid of Snefru at Dahshur. This shows glyphs associated with the left side of a heb-sed scene including half-sky glyphs with shen-rings and the bull's tail. |  | Fakhry, A., The Monuments of Snefru at Dahshur. Volume 2. Part 1. The Temple Reliefs (1961), 138, fig. 170. |


| 4 | 21 | Fragmented part of relief from the Valley Temple of the Bent Pyramid of Snefru at Dahshur. This shows a shen-ring held in the claws of a Horus falcon directly over a cartouche of Snefru. |  | Fakhry, A., The Monuments of Snefru at Dahshur. Volume 2. Part <br> 1. The Temple Reliefs (1961), 144, fig. 191. |
| :---: | :---: | :---: | :---: | :---: |
| 4 | 22 | Fragmented part of a relief from the Valley Temple of the Bent Pyramid of Snefru at Dahshur. Motif of flying Horus falcon holding shen-ring under starry sky band. |  | The Monuments of Snefru at Dahshur. Volume 2. Part 1. The Temple Reliefs (1961), 149, fig. 206. |
| 4 | 23 | Relief fragments from the Valley Temple of the Bent Pyramid of Snefru at Dahshur. Three fragmented flying Horus falcons holding shen-rings, one over a glyph known as the golden Horus. |  | Fakhry, A., The Monuments of Snefru at Dahshur. Volume 2. Part <br> 1. The Temple Reliefs (1961), 151, figs. 214 (left), 215 (above right), 216 (lower right). |
| 4 | 24 | Fragments of a relief from the Valley Temple of the Bent Pyramid of Snefru at Dahshur showing flying Horus falcon holding shen-ring. |  | Fakhry, A., The Monuments of Snefru at Dahshur. Volume 2. Part 1. The Temple Reliefs (1961), 153, fig. 221. |
| 4 | 25 | Fragments of a relief from the Valley Temple of the Bent Pyramid of Snefru at Dahshur showing right side of heb-sed scene including half-sky glyphs. |  | Fakhry, A., The Monuments of Snefru at Dahshur. Volume 2. Part 1. The Temple Reliefs (1961), 168, figs. 273, 274. |
| 4 | 26 | Fragments of a relief from the Valley Temple of the Bent Pyramid of Snefru at Dahshur. This shows glyphs associated with the left side of a heb-sed scene including half-sky glyphs with shen-rings, and the bollards, and the bull tail near the heel of the running pharaoh. |  | Fakhry, A., The Monuments of Snefru at Dahshur. Volume 2. Part <br> 1. The Temple Reliefs (1961), pl. XXVII. |
| 4 | 27 | Relief from the tomb of lynefer, son of Snefru, brother of Khufu. From Dahshur. From northern niche of tomb. Figure is shown in profile looking to right seated on a backless chair with bowed wood supports on the sides. Holds a straight staff in left hand. He has a shen around his neck in a similar arrangement to the Louvre statue of Ankh described above in item 13. | Cairo Museum <br> JE 38564 <br> or <br> CG 57T21. | Alexanian, N., Das Grab des Prinzen Netjeraperef: Die Mastaba II/1 in Dahschur (1999), pls. 18 d -e. |
| 4 | 28 | Tall vertical panel of iconographic decoration on uprights from a wooden bedroom canopy. Belonged to Hetepheres, mother of Khufu and wife of Snefru, includes cartouches of Snefru. Found in tomb G7000 X at Giza. Scene includes three flying Horus falcons with shen-rings over each of three subsections. Top subsection contains Snefru's serekh. Middle contains Snefru's cartouche. | Cairo Museum <br> JE 57711. | Reisner, G.A., BMFA (1932), No.30., 56-60, also Reisner, G.A. and Smith, W., A history of the Giza Necropolis Volume 2. The tomb of Hetepheres Mother of Cheops (1955), pls. 8, 9, \& 10 . |


| 4 | 29 | Decorated curtain box. Belonged to Hetepheres. Found in tomb G7000X at Giza. Text reads "The pharaoh Snefru, protected by the vulture goddess Nekhbet, lady of the sanctuary of Nekhbet at Hierakonpolis. Protected by the cobra goddess Wadjet, lady of the pr-nu shrine of Lower Egypt; Horus, Lord of Ma'at, Snefru, foremost of immortal ka spirits, Snefru, Lord of Ma'at, protected by Nekhbet, given life forever." Lower left end panel contains flying solar disk over cartouche and glyphs for "protection, life around him forever". Opposite end includes Nekhbet with shen-ring and Wadjet providing protection around the pharaoh. Serekh surmounted by Horus faces pharaoh. Cartouche is behind pharaoh. | Cairo Museum <br> JE 72030. | (Figure 4-6); Reisner, G.A. and Smith, W., A history of the Giza Necropolis Volume 2. The tomb of Hetepheres Mother of Cheops (1955), figs. 28a \& 28b above, 29a \& 29b below, pls. 11 \& 12. |
| :---: | :---: | :---: | :---: | :---: |
| 4 | 30 | Gold covered chest belonging to Hetepheres. Text reads "mother of the king of Upper and Lower Egypt; attendant/follower of Horus; controller of the butchers of the acacia house (who prepare sacrifices for funerary rituals); she whose every word is done for her; daughter of the god's body, Hetepheres". | Cairo Museum. | Reisner, G.A. and Smith, W., A history of the Giza Necropolis Volume 2. <br> The tomb of Hetepheres Mother of Cheops (1955), fig. 40. |
| 4 | 31 | Relief showing flying Horus falcon holding the shen-ring. Well detailed claws and rope fibers. From Khufu's pyramid causeway at Giza, reused in Amenemhat l's pyramid complex at Lisht. Text reads "for the shrine of upper and lower Egypt, the house of the great god in the horizon". | MFA Bos- <br> ton 21/58.322. | (Figure 4-8); Goedicke, H., Reused Blocks from the Pyramid of Amenemhat I at Lisht (1971), 14 \& 15 . |
| 4 | 32 | Relief showing shen-ring and a section of the tips of a falcon's wing feathers above Khufu, who wears the red crown. From the southern wall of the causeway linking to Khufu's pyramid on the east side. Shows Khufu on ceremonial visit to Heliopolis wearing special cloak. | Unknown, Possibly in storage in Egypt. | Reisner, G.A. and Smith, W., A history of the Giza Necropolis Volume 2. <br> The tomb of Hetepheres Mother of Cheops (1955), fig.5, and Hassan, S., The Great Pyramid of Khufu and its Mortuary Chapel (1960), pl. vi \& fig. 4. |
| 4 | 33 | Greywacke triad statue of Menkaure with Hathor, and the Cynopolis nome goddess with Anubis/ jackal insignia above head. Discovered in Menkaure's valley temple by G. Reisner. Hathor holds shen-ring in right hand, nome goddess holds shenring in left hand. These are the only known threedimensional renderings of shen-rings dating to the Old Kingdom. | Cairo Museum <br> JE 40679 (often confused with JE 46499). | (Figure 4-15) |
| 4 | 34 | Pair of finely detailed panels on either side of the base of a travertine/Egyptian alabaster seated statue of Menkaure. The upper part of the body is now lost. Panels include royal insignia with paired Horus falcons with shen-rings in flight above, under sky motif. Prominent $s m 3$ - $t 3$ wi motifs on either side. Iconography refers to Upper Egypt on left side, and Lower Egypt on right side. | MFA Boston 09.202. | (Figures 4-12 and 4-13); C.f. MFA website. |


| 5 | 35 | Elaborate relief from the pyramid of Userkaf reused in the pyramid of Amenemhet I at Lisht above robbers' tunnel. Found in 1991 by MMA expedition. Possibly unfinished scene of flying falcon surrounded by symbols. Nekhbet and Wadjet hold shen-rings and $w 3 s$ scepters to the pharaoh's names. The text reads, top: "The Behdite (Horus), great god of multi-colored plumage who comes forth from the horizon". Left: "Wadjet, lord of the per-nu shrine of Lower Egypt, of Pe/Pt (Buto)". Center: "Lord who made all things, king of Upper and Lower Egypt, Userkaf, Horus who makes Ma'at, give him life". Right: "Beautiful god, lord of appearances, the shining one". Bottom: "Give him life, authority, power, all joy, and health forever". | NY Met N.A. 1992.2. | (Figure 5-1); See Allen, J.P. et al. (eds.), Egyptian Art in the Age of the Pyramids (1999), 319 \& 321. |
| :---: | :---: | :---: | :---: | :---: |
| 5 | 36 | Panel on east side of a red granite column on the north side of the central courtyard at the entrance to the pyramid temple of Sahure at Abusir. Shows cartouche of Sahure and serekh surmounted by Horus wearing red Deshret crown of Lower Egypt. Horus faces north to Wadjet goddess of the Delta who holds forth a shen-ring and the $w 3 s$ scepter emblems. |  | (Figure 5-2); Borchardt, L., Das Grabdenkmal des Königs S'aḥu-Re (Band I): Der Bau (1910), 44 \& 45. |
| 5 | 37 | Panel on the east side of the rose granite column on the south side of the central courtyard at the entrance to the pyramid temple of Sahure at Abusir. Shows cartouche of Sahure and serekh surmounted by Horus wearing white Hedjet crown of Upper Egypt. Horus faces south to Nekhbet; goddess of Upper Egypt who holds forth a shen-ring and the $w 3 s$ scepter emblems. |  | (Figure 5-2); Borchardt, L., Das Grabdenkmal des Königs S'aḥu-Re (Band I): Der Bau (1910), 44 \& 45. |
| 5 | 38 | Lintel block from the Pyramid of Sahure at Abusir. Vulture Nekhbet and uraeus Wadjet goddesses of north and south with shens-rings and w3s scepters face the shrines and towns of Upper and Lower Egypt. |  | Borchardt, L., Das Grabdenkmal des Königs S'aḥu-Re (Band I): Der Bau (1910), pl. 10. |
| 5 | 39 | Elaborate relief from the pyramid complex of Sahure. Vulture with shen-ring flies beneath starry sky above sphinx trampling enemies. Text reads "who binds the (nine) bows (enemies of Egypt), she is the lady of the palace of Upper Egypt. She encircles and protects him, lord of the two lands, the pharaoh Sahure. He is Horus, strong armed, who acts with his own hand. Lord of the two lands, given all life and stability, all health, all happiness, all joy. He is the foremost of the immortal ka spirits. He is Thoth? lord of the Nubian nomads of the south, he is Sopdu, Lord of foreign lands in the east, he tramples the Libyan rebel chiefs from the west. He is given life, all power". |  | Borchardt, L., Das Grabdenkmal des Königs S'aḥu-Re (Band II): Die Wandbilder (1910), pl. 8. |
| 5 | 40 | Fragments of reliefs from the pyramid of Sahure including a shen included in a column of hieroglyphs, and another two shown as parts of half-sky signs from the side of a heb-sed ritual scene. |  | Borchardt, L., Das Grabdenkmal des Königs S'aḥu-Re (Band II): Die Wandbilder (1910), pl. 46. |


| 5 | 41 | Fragmented relief scene of flying vulture with shen-ring from the pyramid of Sahure. |  | Borchardt, L., Das Grabdenkmal des Königs S'aḥu-Re (Band II): Die Wandbilder (1910), pl. 64. |
| :---: | :---: | :---: | :---: | :---: |
| 5 | 42 | Fragments of a relief showing vulture with shenring and enigmatic cloaked figure on dais. From the pyramid complex of Sahure. |  | Borchardt, L., Das Grabdenkmal des Königs S'aḥu-Re (Band II): Die Wandbilder (1910), pl. 65. |
| 5 | 43 | Fragments of a relief from the pyramid complex of Sahure showing flying falcon Horus holding shen in claws under starry sky motif. Also, second fragment of similar avian deity possibly flying falcon with shen in claws. |  | Borchardt, L., Das Grabdenkmal des Königs S'aḥu-Re (Band II): Die Wandbilder (1910), pl. 66. |
| 5 | 44 | Fragments of relief from the pyramid complex of Sahure showing pharaonic iconography and glyphs including serekhs of Sahure surmounted by Horus, Wadjet cobra with shen supported by papyrus plants of Lower Egypt, and associated hieroglyphs. |  | Borchardt, L., Das Grabdenkmal des Königs S'aḥu-Re (Band II): Die Wandbilder (1910), pl. 69. |
| 5 | 45 | Relief fragments from the pyramid complex of Sahure showing Wadjet cobra with shen and w3s scepter, shrine of Lower Egypt per-nu and papyrus plant emblems of Lower Egypt also shown. Possible rings above uraeus snake. |  | Borchardt, L. Das Grabdenkmal des Königs S'aḥu-Re (Band II): Die Wandbilder (1910), pl. 70. |
| 5 | 46 | Tall sycamore wood $h s$ offering jar with faience inlays and gold leaf detailing. Inlays simulate falcon feathers. From funerary temple of pyramid of Neferirkare at Abusir. Reconstruction based on fragments collected during excavations. Form and decoration represent a falcon's body and folded wings; wd $3 t$-eye on shoulder. Shen-rings alternate with symbols of god Min on a band around the waist of the jar. Height 45 cm . | $\begin{aligned} & \hline \text { Berlin ÄM } \\ & 18807 . \end{aligned}$ | (Figure 5-6); See Allen J.P. et al. (eds.), Egyptian Art in the Age of the Pyramids (1999), 344-347; Borchardt, L. MDOG No. 34 (1907), 37-39, Blatt 3. |
| 5 | 47 | Rose limestone statue of the pharaoh Neferefre from his mortuary temple at Abusir. Discovered during excavations by Verner in 1984. Horus stands behind pharaoh's head holding wings around the sides protectively, and with shen-rings in claws. C.f. green gneiss statue of Khafre enthroned in Cairo Museum (JE 10062). | Cairo Museum <br> JE 98171. | (Figure 5-5); Verner, M., Abusir: Realm of Osiris (2002), 112 \& 129. |
| 5 | 48 | Fragmented relief section from the pyramid of Niuserre at Abusir. Shows flying vulture goddess with large round shen-ring in claws. | Berlin <br> ÄM 16102. | Borchardt, L., Das Grabdenkmal des Königs Ne-User-Re, Ausgrabungen der Deutschen Orientgesellschaft in Abusir 19021904 (1907), 88 \& fig. 67 on 89. |
| 5 | 49 | Travertine/Egyptian alabaster jar with panel inscribed for Djedkare Isezi. Nekhbet and Wadjet holding shen-rings flank the cartouche and serekh of the pharaoh. Main text reads "Given life forever, beloved". Additional text reads "now in the time of Isesi, royal adornment (for every) 10 days, ibu oil, $3 / 4$ of a dut measure". | British Museum EA 57322 | Strudwick, N., Texts of the Pyramid Age (2005), 129; Spieser, C., Les Noms du Pharaon (2000), 380, fig. 302. |


| 5 | 50 | Horizontal limestone block of stone depicting a large offering table/hotep glyph on the upper surface. Flanked by two large shen-rings where the offering jars or 'reversion offerings' were typically placed. At the entrance to the Tomb of Khuwy, a senior official during reign of Djedkare Isesi. Discovered south Saqqara 2019. | In situ. |  |
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| 5 | 51 | Decorated spheroid travertine/alabaster jar dedicated to pharaoh Unas. In the form of a $n w$-vase. The choice of travertine/alabaster material for this group of vases is likely intentional to resemble ostrich eggs. The iconography represents a Horus falcon encircling the pharaoh's prenomen in a cartouche. Designs take advantage of natural bands of lighter stone in the matrix of the travertine/Egyptian alabaster. Light band encircles main falcon on front of vessel. A second encircling band runs around reverse. Falcon holds two shen-rings in claws. Provenance unknown. Height 16 cm . | Louvre Museum $\text { E } 32372 .$ | (Figures 5-7 \& 5-8); See Ziegler, C. in Egyptian Art in the Age of the Pyramids, Allen, J.P. et al. (eds.), (1999), 361. |
| 5 | 52 | Decorated globular travertine/Egyptian alabaster jar from the reign of Unas. Motif includes two large falcons with spread wings holding shen-rings in their claws. Panel between them on front side includes a cartouche and serekh of the pharaoh and a formulaic dedicatory inscription, and on the reverse side is a single shen-ring in isolation. Translucent band runs around vase and directly through iconography. Height 26.2 cm . | Oriental Institute Chicago IM13947. | (Figures 5-9 \& 5-10); Bailleul-LeSuer, R., Between Heaven and Earth: Birds in Ancient Egypt (2012), 206. |
| 5 | 53 | Abusir Papyri. Shen-rings appear on pls. 31 (fragment B.1), 31A, 27F, 29E, 32.7, and 67 (fragment B) |  | Posener-Kriéger, P., The Abu Sir Papyri. Hieratic Papyri in the British Museum. Fifth Series (1968). |
| 6 | 54 | Line of text from a biographical inscription on the mastaba of Hnw , Saqqara. The tomb is unnumbered. The last part of the inscription is the name of Hnw's son, S -n-Wnis. The text indicates that he was a priest at the pyramid of Unas. A glyph shows the priest with an offering tray with a shen-ring below, possibly indicating that these are "reversion" offerings. Possibly to be taken to the pyramid's offering table as part of the daily ritual encircling rounds. |  | PM III Memphis, 2 Saqqara to Dahshur, 625. |
| 6 | 55 | Fine belt buckle belonging to son of pharaoh Pepi II, prince Ptahshepses. Gold inlaid with faience, soap stone, and carnelian. Attached to obsidian and turquoise beaded gold belt. Found in the bandages of the prince's mummy at Saqqara in the lower temple of Unas's complex, in the sarcophagus of Ptahshepses (1944). Two flying Horus falcons with shen-rings face outwards towards the enthroned figures of the deceased, which are repeated on either side of the buckle facing inwards. The encirclement theme matches the function of the belt. Similar encircling iconography is found on bracelets and pectorals in later eras, which had protective value. | Cairo Museum <br> JE 87078. | Dodson, A. and Hilton, D., The Complete Royal Families of Ancient Egypt (2004), 78. |


| 6 | 56 | Travertine/Egyptian alabaster jar with inscription dedicated to pharaoh Teti from Tell-Edfu. Long encircling band of text reads: "Live, Horus who satisfies the two lands, king of Upper and Lower Egypt, Teti, son of Re, given life, stability, strength, forever". The small rectangular panel with the pharaoh's serekh and cartouche reads: "Horus, king of Upper and Lower Egypt, given life forever". Main motif includes Horus carrying shen-rings in claws, a central lotus flower on the base. Head is missing but can be confirmed as a falcon by tail shape which is narrow and not fanned like vulture. |  | Bruyere, B., Tell-Edfou. <br> Fouilles franco-polonaises, Rapports, 1.' <br> (1937), 35, pls. xvii \& xxii. |
| :---: | :---: | :---: | :---: | :---: |
| 6 | 57 | Jar made from an ostrich egg, from the Mastaba of governor Khentika, Dakhla Oasis in the Western Desert. Found with female relative. May be a rare early example of the iconography being used indiscreetly, outside of the pharaonic context. Horus falcons spreads wings around vase, holds shenrings in both claws. An additional shen is etched around the opening at the top of the egg. A stopper may have been inserted there. | Cairo Museum <br> JE 98774. | Cherpion, N., "L’oeuf d'autruche du mastaba III" in Castel, G. (ed.), Le mastaba de Khentika (2001), balat v, annexe ii, 279-294. |
| 6 | 58 | Fragments of a relief scene of Queen Iput I, daughter of Unas, wife of Teti and mother of Pepi I Meryre. From her pyramid chapel at Saqqara. Horus falcon with shen-ring flies over queen who wears vulture headdress. Starry sky band overhead. |  | Dodson, A. and Hilton, D., The Complete Royal Families of Ancient Egypt (2004), 67. |
| 6 | 59 | Travertine/Egyptian alabaster jar inscribed with rectangular panel for Pepi I celebrating his first sed festival and mentioning his pyramid at Saqqara. Shows priest with offering table of vessels. Shen sign and ankh sign under table. Similar to representation from mastaba of Hnw shown above. May represent "reversion" offerings to be taken to the offering table at the pyramid by a circuitous ritual route. Sky band over top of arrangement, w3s scepters to either side. Rest of jar is undecorated. | Egyptian <br> Museum Berlin <br> ÄM 7715. | http://www.smb-digital. de/eMuseumPlus?servic e=ExternalInterface\&mo dule=collection\&object\| d=761358\&viewType=de tailView |
| 6 | 60 | Stone weight from the reign of Pepi I Meryre, decorated with panel containing royal symbols of Upper and Lower Egypt, and including Wadjet with shen-ring on papyrus plant of Lower Egypt. Formulaic arrangement of symbols on a small scale. | Oriental Museum of the University of Durham, EG457. |  |
| 6 | 61 | Fragment of sculpted vase with carved and painted falcon's head and painted body. <br> Head is prehensile element with decorative and functional purpose. Falcon's eye markings are well defined. One claw is visible holding shen-ring. From queen Merietites IV's pyramid temple near husband Pepi I's pyramid at Saqqara. She was also a pharaoh's daughter. |  | Leclant, J. and Clerc, G., "Fouilles et travaux en Égypte et au Soudan, 1994-1995" in Orientalia (1996), 234-356 and Labrousse, A., Les pyramides des reines: Une nouvelle necropole a Saqqara (1999). |


| 6 | 62 | Rock-cut relief dedicated to pharaoh Pepi I Meryre in Wadi Maghara near the turquoise mining area, southwest Sinai Peninsula. Left of tripartite scene shows pharaoh in smiting pose with winged solar Behdite disk above. Middle scene shows pharaoh in ritual running pose under flying Horus falcon holding shen-ring. Text behind pharaoh states: "protection and life around him". Serekh surmounted by Horus on right of panel. | Lepsius, R.C., Denkmäler aus Aegypten und Aethiopien, plates, Old Kingdom (1850), part 2, band 3, fig. 116a. |
| :---: | :---: | :---: | :---: |
| 6 | 63 | Fragment of relief showing wadjet holding w3s scepter and shen-ring towards golden Horus name and cartouche of Pepi II, including Son of Re title, resting on papyrus plant of Lower Egypt. Found east side of the pyramid of Amenemhet at Lisht. | Goedicke, H., Reused Blocks from the Pyramid of Amenemhat I at Lisht (1971), 27 \& 28. From unknown monument of Pepi II. Cat No. L 6-7:317. |
| 6 | 64 | Relief fragment from the mortuary temple of the pyramid of Pepi II at Saqqara. Finely detailed flying vulture goddess holds shen-ring in claws. | Jéquier, G., Le monument funéraire de Pépi II: Le temple, Vol. 2 (1938), pls. 32 \& 33. |
| 6 | 65 | Fragmented relief scene from the mortuary temple of the pyramid of Pepi II at Saqqara. Flying vulture goddess holds shen-ring in claws; djed and ankh behind. | Jéquier, G., Le monument funéraire de Pépi II: Le temple, Vol. 2 (1938), pls. 46 \& 47. |
| 6 | 66 | Fragmented relief scene from the mortuary temple of the pyramid of Pepi II at Saqqara. Flying Horus falcon holds shen-ring in claws over nswtbity title. Text above refers to row of shrines, perhaps part of heb-sed ritual. | Jéquier, G., Le monument funéraire de Pépi II: Le temple, Vol. 2, (1938), pls. 50 \& 51. Also see Allen, J.P. et al. (eds.), Egyptian Art in the Age of the Pyramids, (1999), 88, fig. 52. |
| 6 | 67 | Fragments of relief scene from the mortuary temple of the pyramid of Pepi II at Saqqara. Starry band across top. Part of text refers to row of shrines. Falcon holds shen-ring over nswt-bity title and Horus of gold sign. | Jéquier, G., Le monument funéraire de Pépi II: Le temple, Vol. 2, (1938), 61 \& 63 . |
| 6 | 68 | Fragments of an elaborate relief scene from the mortuary temple of the pyramid of Pepi II at Saqqara. Horus with double crown stands on serekh. Wadjet goddess of Lower Egypt stands on papyrus plants holds shen-ring and $w 3 s$ scepter to beak of falcon. Shrines including pr-nw shown. nswt-bity title behind falcon. | Jéquier, G., Le monument funéraire de Pépi II: Le temple, Vol. 2 (1938), pls. 81, 83, 84. |
| 6 | 69 | Fragments of relief scene from the mortuary temple of the pyramid of Pepi II at Saqqara. Shows part of decorated serving bowl overflowing with lotus lilies and other foliage. Iconography shown on bowl includes cartouche, pair of opposing falcons flanking either side of bowl holding shenrings, $n s w t$-bity title and "life forever" included. There is no comparanda for this bowl, which was reconstructed by the author in 2016. | (Figure 6-1); Jéquier, G., Le monument funéraire de Pépi II: Le temple, Vol. 2 (1938), pl. 104. |


| 6 | 70 | Carved wooden panel adorned with gold leaf from <br> the mortuary temple of the pyramid of Pepi II at <br> Saqqara. The iconography includes a prominent <br> central cartouche, two opposing flanking Behdite <br> falcons wearing double crowns and holding shen- <br> rings. |  | Jéquier, G., Le monument <br> funéraire de Pépi II: Le <br> temple, Vol. 2 (1938), 39 <br> $\&$ 40, fig 28, see also 6, <br> item 8 for description. |
| :--- | :--- | :--- | :--- | :--- |
| 6 | 71 | Fine travertine/Egyptian alabaster statue of queen <br> Ankhnes-meryre II who acted as regent, with <br> young pharaoh Pepi II on knee. A pair of shen- <br> rings are held on either side of her head by a vul- <br> ture goddess. Queen shown as protectress of the <br> young pharaoh. Find spot unknown. | Brooklyn <br> Museum, <br> New York. <br> 39.119. | C.f. museum website. |

References that are contained in this table but not used in the rest of the text are listed in the footnote below and in the bibliography. ${ }^{201}$

## Addendum: A mathematical note


#### Abstract

This addendum addresses a specific issue in Egyptian mathematics that merits special discussion. Due to its complexity, it has been separated out from the main body of this thesis into this short article. The proposal made here is that calculations involving circles in ancient Egyptian papyri ${ }^{202}$ can elucidate the early development of ancient Egyptian mathematics, as well as the architectonic tradition of encircling protective symbolism discussed in the current work. ${ }^{203}$ This elucidation can only be achieved, however, when the information is rigorously interpreted. The subject has remained confused for many decades, ${ }^{204}$ and has at times even been contentious, ${ }^{205}$ and the technical discussions have lacked clarity ${ }^{206}$ for a number of different reasons. In order to clarify the issue, the main objective of this addendum is to show the importance of clearly differentiating between calculations involving circular areas, and calculations involving the lengths of circumferences, during this early period of mathematical development.


A handful written examples (Figure A-1) involving circles have been recovered from ancient Egyptian mathematical papyri. ${ }^{207}$ They include a method ${ }^{208}$ to calculate the area of a circle by using its width. The algorithm seemingly equates the circular area to that of a similar square. In problem 41 on the Rhind Mathematical Papyrus, a factor of $8 / 9$ is first applied to the width 9 of the circle given in the example, and the result is then multiplied by itself. By multiplying the factor by itself it is effectively squared, and this gives a resulting area that is $64 / 81$ times that of a 9 x 9 square that would contain the circle in question. The result is an accurate approximation of the circular area contained within that square, being only $0.6 \%$ in excess of the actual value. This is a very different procedure to one employing a radius or a rotated diameter to produce a circumference, or that uses

[^29]a constant ratio to relate radii or diameters numerically to circular areas. ${ }^{209}$ Unfortunately, these texts have been construed by some scholars as evidence indicating that the ancient Egyptians could not calculate the lengths of circumferences, and that they certainly did not know that a circumference is $3+1 / 7$ th times the length of a diameter (a factor now expressed as $22 / 7$ ). It is important to understand, ${ }^{210}$ however, that these written examples are calculations of areas, not circumferences, and so the existence of such an algorithm does not preclude the existence of other methods used for calculating the lengths of circumferences. The attested area method ${ }^{211}$ does produce a value that is approximately equal to the area of the circle of the specified width, but there is nothing in these problems that relates to circumferences at all. Furthermore, there is nothing in these problems to suggest that the ancient Egyptians were aware that circular area and circumference calculations can be related, as we do today with one common factor now known as pi. ${ }^{212}$


Example of making a granary, round by 9 , by 10 (tall)
Fig. A-1. Transcription of the rubric text for problem 41 on the Rhind mathematical papyrus. The hieratic, hieroglyphs, transliteration (right to left then left to right), and translation are given. The example as a whole demonstrates how to calculate the volume of a cylindrical granary and does not include reference to a circumference. The area of the circular section is first calculated using an area algorithm and the result is then multiplied by the height to obtain the volume. The image above shows the first line of the text stating the problem to be solved. This part of the text is in red, apart from the number 10 , which may have been added retrospectively after the scribe Ahmes had switched inks. The lines of hieratic symbols and hieroglyphs should be read from right to left.

The algorithms relating to circular areas on the papyri then, do not preclude the existence of a different calculation method that utilized lengths of radii or diameters of circles to calculate the lengths of circumferences. While there is no written evidence of such a method in the recovered papyri, several scholars have noted that architectural evidence from the ancient Egyptian monuments indicates that such a circumference calculation method was known, ${ }^{213}$ at least during the Old Kingdom.

[^30]As set out in the current work, analysis of Old Kingdom pyramids indicates that a circumference algorithm was known that used a length of 7 parts as the radius or diameter of a circle. The circumference was then 44 parts if the radius was used to construct the circle, or 22 parts if the diameter was used. These basic numbers work readily within the ancient Egyptians' 7-part cubit metrical system and are very accurate (Figure 4-7). ${ }^{214}$ While there is nothing to show that this value was understood abstractly as a ratio, a basic knowledge of these numbers could have been used to scale-up geometric proportions to whatever architectural dimensions were required, using simple multiplication. This certainly does not constitute a full-blown understanding of the pi ratio, but if the method was used then it did serve as a precursor version of the pi ratio, at least with respect to circumferences if not areas. There is no evidence in the texts, nor in the architectural data, to suggest that such circumference related numbers were somehow adapted for use in circular area calculations. Evidence of that level of understanding is only present in documents created many centuries later.

The argument made here may seem like splitting hairs, but when dealing with the field of pure mathematics and its early development, rigorous logic is vitally important and allows us to draw appropriate conclusions from the limited evidence available. When carefully dissected, it seems that the textual and archaeological bodies of evidence are in fact entirely compatible rather than contradictory, and this consistency should be expected if the historical record is being interpreted correctly.

Like many of the other examples on the papyri the approximate circular area calculation method described was most likely intended for agricultural and alimentary quantification purposes (fields areas, granary volumes). ${ }^{215}$ More accurate construction processes involving circumference may then have been reserved for setting out side-lengths and perimeters of monumental structures. There may have been different methods used in different trades, and so quite different and unrelated methods could have developed in parallel over time, particularly if the algorithms had two different mathematical functions that were only found to be relatable many centuries later. ${ }^{216}$

Another argument put forward by skeptics was that the papyri show examples of the seked slope measurement system being used to define the side slopes of pyramids, rather than any geometry related to the proportions of the monument. In fact, the examples show that the seked was derived from the base lengths and heights, already selected by the architects, but the factors that influenced those choices are not addressed in the examples. By following the questionable logic of the skeptics to an end, the authors of those discussions were forced to conclude that "taking the pyramids as a whole, it seems that the architects were not particularly concerned about the exact height" ${ }^{217}$ As the archaeological surveys have shown, however, it is very clear that the Old Kingdom Egyptians were very interested in the ultimate heights of their monuments and could measure and build them with great accuracy, and so a more appropriate consideration of the engineering, architectonic, and symbolic factors that influenced their height and base length choices was certainly required.

Several Egyptologists, perhaps most notably Petrie, ${ }^{218}$ concluded that numbers and

214 Other researchers have noted the repeated occurrences of the numbers $7,11,22$, and 44 in other aspects of the architecture of the early Old Kingdom pyramids. C.f. L. Miatello, "A 'Solar Rule' in the Architecture of Fourth Dynasty Pyramids", Ankh 17 (2008). He also related these numbers to circles and the solar circle in particular.

215 Examples on the P. Cairo. J.E. 89127-30, 89137-43 dating to the 3rd century B.C. include problems 32 and 33 that incorporate diameters, circumferences, and circular areas for the first time. See R.A Parker, Demotic Mathematical Papyrii (Rhode Island: Brown University Press, 1972), 40-52.
216 E. Zapassky et al., "An Ancient Relation between Units of Length and Volume Based on a Sphere", PloS One 7, no. 3 (2012).
217 Robins and Shute (1985), 112.
218 Petrie (1940), 76; "Surveys of the Great Pyramids", Nature (1925); (1892), 30; idem, The Pyramids and Temples of Gizeh - 2nd Edition from 1885 Republished in a New and Revised Edition with an Update by Zahi Hawass (London: Histories and Mysteries of Man Ltd, 1990).
proportions relating to circles were incorporated into the primary dimensions of the Great Pyramid and the pyramid of Meidum. It seems most likely to the current author that this architectonic tradition was a means of bestowing enduring encircling symbolic protection on the monuments, and that this was represented graphically by the shen-ring. This concept was first incorporated into a pyramid complex at Saqqara, where the Step Pyramid's exterior perimeter, a monumental temenos wall, is the length of a circle of diameter 1000 cubits. ${ }^{219}$

Additional evidence exists indicating the presence of a well-developed numerical understanding of circles in the architectural domain during the Old Kingdom. This includes the use of fine embedded circular columns decorating the entrance to "T temple" of the Step Pyramid at Saqqara, which would have had 22 channels running down their faces at equal intervals if completed in the round. It required a fairly sophisticated geometric understanding of circumferences to manufacture them accurately, and so their subdivision in that way is notable. ${ }^{220}$

The basic hypothesis here is that simple but accurate architectural methods existed during the Old Kingdom that were used for calculating circumferences and that were applied within the domain of monumental architecture. The methods do not appear on the later papyri and were unrelated to the ancient Egyptian methods for calculating circular areas. This means that the examples in the papyri provide evidence complimentary to the architectural evidence, rather than contradictory to it. ${ }^{221}$ That in itself is a significant conclusion to draw and a small step forwards in our understanding of the historical development of mathematics and architecture in the ancient world.

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[^0]:    1 D. Lightbody, Egyptian Tomb Architecture. The Archaeological Facts of Pharaonic Circular Symbolism, vol. S1852 (Oxford: Archaeopress British Archaeological Reports International Series, 2008).
    2 Proportion is not synonymous with ratio. A proportion in art or architecture refers to the relative spatial magnitudes of elements of the composition. The related magnitudes can be expressed as a numerical ratio when their lengths, areas, or volumes are calculated or measured as numerical values and compared. The proportional relationship is not fixed to particular magnitudes or dimensions, and can be reproduced at different scales.
    3 In order to help develop an open forum where papers focusing on ancient Egyptian architecture could be submitted, peerreviewed, published, and discussed, I worked with Franck Monnier to establish the Journal of Ancient Egyptian Architecture in 2016, and we were able to do this with the help of several professional and amateur colleagues. Paul François designed a bespoke online reading interface to make the peer reviewed studies available world-wide. See: The Journal of Ancient Egyptian Architecture online at www.Egyptian-architecture.com. We continue to maintain the website and associated social-media forum and volume 4 will be published in 2020.

[^1]:    8 The great man theory is a 19th century concept according to which history can be largely explained by the impact of great men, heroes, or other highly influential individuals who, due to either their personal charisma, intelligence, wisdom, or political skill, used their power in a way that had a decisive historical impact. The theory was popularized in the 1840s by Scottish writer Thomas Carlyle.
    9 One of the first to critique Carlyle's proposal was English intellectual Herbert Spencer.
    10 Fernand Braudel's Longue Durée is the most notable of these approaches.
    11 This is made particularly difficult by the fact that ancient Egyptian artisans did not sign their work.

[^2]:    12 J. Baines, "On the Status and Purposes of Ancient Egyptian Art", CAJ 4:1 (1994), 68.
    13 Ibid., 77.
    14 Ibid., 72.
    15 J. Baines, "Communication and Display: The Integration of Early Egyptian Art and Writing", Antiquity 63 (1989), 480.
    16 R.H. Wilkinson, Symbol and Magic in Egyptian Art (New York: Thames and Hudson, 1999), 7.
    17 R.K. Ritner, The Mechanics of Ancient Egyptian Magic, Studies in Ancient Oriental Civilization No. 54 (Chicago: The Oriental Institute of the University of Chicago, 2008), 8.
    18 J. Baines, "Temples as Symbols, Guarantors and Participants in Egyptian Civilization", in The Temple in Ancient Egypt: New Discoveries and Recent Research, ed. S. Quirke (London: British Museum Press, 1997), 217.
    19 A similar concept, the "cognitive constellation", has been described by Renfrew. See C. Renfrew, "Symbol before Concept: Material Engagement and the Early Development of Society", in Archaeological Theory Today, ed. I. Hodder (Cambridge: Cambridge University Press, 2001), 137. He described how, in sedentary societies, symbols come to constitute reality in rituals and religions, as much as they reflect them.
    20 Baines, "Temples as Symbols, Guarantors and Participants in Egyptian Civilization", 217.

[^3]:    21 Baines, "Communication and Display: The Integration of Early Egyptian Art and Writing", 479.
    22 See Addendum. A mathematical note, at the end of this work, 80.
    23 J.P. Allen, Middle Egyptian, an Introduction to the Language and Culture of Hieroglyphs (Cambridge: Cambridge University Press, 2001), 65; S. Quirke, The Cult of Ra. Sun Worship in Ancient Egypt from the Pyramids to Cleopatra (London: Thames \& Hudson Ltd., 2001), 123; A. Gardiner, Egyptian Grammar: Being an Introduction to the Study of Hieroglyphs, 3rd Rev. (London: Oxford University Press, 1957), 74.
    24 W. Barta, "Der Königsring als Symbol zyklischer Wiederkehr", ZÄS 98 (1970); A.O. Bolshakov, Man and His Double in Egyptian Ideology of the Old Kingdom, Ägypten Und Altes Testament (Wiesbaden: Harrassowitz Verlag, 1997); A. Sugi, "The Iconographical Representations of the Sun God in New Kingdom Egypt", in Egyptology at the Dawn of the Twenty-First Century. Proceedings of the Eighth International Congress of Egyptologists, Cairo 2000. Vol 2. History, Religion, eds. Z. Hawass and L.P. Brock (Cairo AUC, 2003); R.H. Wilkinson, Reading Egyptian Art. A Hieroglyphic Guide to Ancient Egyptian Painting and Sculpture (London: Thames and Hudson Ltd., 2003), 192-94; C. Spieser, "Cartouche", in the UCLA Encyclopedia of Egyptology (Electronic Resource), eds. E. Frood and W. Wendrich (Los Angeles: UCLA, 2010); idem, Les Noms Du Pharaon: Comme Etres Autonomes Au Nouvel Empire (Fribourg: Editions Universitaires Vandenhoeck \& Ruprecht Göttingen, 2000); D.J.O. Klop, Beneath the Raptor's Wings: The Avian Composition Grasping the Symbol for Eternity in Egypt. MPhil Thesis (Stellenbosch: University of Stellenbosch, 2008); L. Miatello, "Expressing the Eternity as Seriality: On šn as a Number of Large Magnitude", JARCE 52 (2016).
    25 M.C. Betro, Hieroglyphics. The Writings of Ancient Egypt (New York: Abbeville Press, 1996), 195; I. Shaw and P. Nicholson, The British Museum Dictionary of Ancient Egypt (London: British Museum Press, 1995), 267, 300, 301; Allen, Middle Egyptian, an Introduction to the Language and Culture of Hieroglyphs, 67; Quirke, The Cult of Ra. Sun Worship in Ancient Egypt from the Pyramids to Cleopatra, 123.

[^4]:    38 Penn Museum E 3958. J.E. Quibell, Hierakonpolis I. With a Note by W.M.F. Petrie (London: British School of Archaeology in Egypt, 1900), p. 11, pls. 36, 37, 38; D.P. Silverman, ed. Searching for Ancient Egypt: Art, Architecture, and Artefacts from the University of Pennsylvania Museum (Dallas Museum of Art and Cornell University Press, 1997), 94.
    39 Quibell, 11.
    40 Anthes, 86-89.
    41 Now in Cairo EM, CG 14724.
    42 Now in the Ashmolean Museum collection: AN1896-1908 E.117; Quibell, 11, pl. 37.
    43 J.P. Lauer and P. Lacau, Inscriptions Gravées Sur Les Vases 2: La Pyramide À Degrés., vol. Fouilles à Saqqarah IV (Cairo: IFAO, 1961), 3, no. 18 and pl. 3.
    44 S. Hendrickx, R. Friedman, and M. Eyckerman, "Early Falcons", in Vorspann Oder Formative Phase? Agypten Und Der Vordere Orient 3500-2700 V. Chr., eds. L.D Morenz and R. Kuhn (Wiesbaden: Harrassowitz Verlag, 2011), 132, 135.

[^5]:    45 J.P. Lauer, Saqqara. The Royal Cemetery of Memphis (London: Thames and Hudson, 1976), 76.
    46 This hypothesis is based in part on a private discussion between the author and S. Hendrickx in 2018.
    47 Renfrew described how a "nexus of symbolic concepts" can develop in sedentary societies as they increasingly value precious goods for exchange and express ideological aspirations through monuments. See Renfrew, 131, 137.
    48 Based on the research carried out by J.P. Lauer, the final enclosure wall was significantly enlarged from an earlier and simpler wall, see Lauer, J.P. "Sur certaines modifications et extensions apportées au complexe funéraire de Djoser au cours de son règne" in Pyramid Studies and Other Essays Presented to I.E.S. Edwards, eds. J. Baines, T.G.H. James, A. Leahy, and A.F. Shore. (London: EES, 1988), 11. He wrote: "Cette explication tendrait à confirmer qu'une première enciente plus simple, édifieé pour le mastaba initial M1-M2-M3, aurait été remplacée par l'enceinte bastionnée:..". In English: "This explanation would tend to confirm that a simpler first stage enclosure, built for the initial mastaba $\mathrm{M} 1-\mathrm{M} 2-\mathrm{M} 3$, would have been replaced by

[^6]:    the bastioned enclosure:..". This work discussed and amended earlier observations made by W. Kaiser, "Zu den königlichen Talberzirken der 1. und 2. Dynastie in Abydos und zur Baugeschichte des Djoser-Grabmals", MDAIK 25 (1969), 1-21.
    49 M. Verner, The Pyramids: Their Archaeology and History (London: Atlantic Books, 2003), 461.
    50 B. Kemp, Ancient Egypt. Anatomy of a Civilisation (Oxford: Routledge, 2005), 103.
    51 J.P. Lauer, "Étude sur quelques monuments de la Ille dynastie (la pyramide à degrés de Saqqarah)", Annales du Service des Antiquités de L’Egypte 31 (Cairo: IFAO, 1931), 60.
    52 See D. Arnold, The Encyclopaedia of Ancient Egyptian Architecture (New York: I.B. Taurus, 2003), 61, who gives a range of 52.352.5 cm for the cubit of the Old Kingdom.

    53 In light of the potential significance of the symbolic form of the Saqqara perimeter wall, it is interesting to note that Petrie applied to survey the perimeter wall of Saqqara directly after he had finished his survey of the Giza plateau and its three great pyramids. The survey of Giza was the crucial and accurate survey in which he first concluded that the circular proportions he measured there were deliberately incorporated into the architecture, and were, therefore, a real design principle used by the ancient Egyptians. It seems possible that Petrie then also considered the possibility that the wall at Saqqara could have been significant in this respect, from a symbolic architectural point of view, and he certainly seems to have been keen to excavate and survey there. An 1883 Egyptian Exploration Fund proposal reports that: "In Saqqara, he would examine the peribolus wall of the Step Pyramid to see if it threw light on the date of the pyramid; this would take a week", In fact, Petrie tried time and time again to obtain a permit to dig at Saqqara and it remained one of the few major sites in Egypt never to have come under his spade. M.S. Drower, Flinders Petrie: A Life in Archaeology (London: Victor Gollancz Ltd., 1985), 69, 272.

[^7]:    Fassadengliederung der grosen Mastabagraber aus der 1. Dynastie", MDAIK 47 (1991), 83.
    56 J. Baines and J. Malek, Atlas of Ancient Egypt (Cultural Atlas) (New York: Facts on File Inc., 1980), 110; Kemp, 112; J.P. Lauer, "Observations sur les pyramides", (Cairo: IFAO, 1960), 2.
    57 Stadelmann described the enlarged wall as part of an effort to create a more "harmonious ensemble": Stadelmann, R. "Origins and Development of the Funerary Complex of Djoser" in Studies in Honor of William Kelly Simpson, edited by P. Der Manuelian (Boston: MFA, 1996), 787; one that was built at an increased size to give an impression of grandeur and harmony, ibid., 798; the changes made were a visible expression of an eternal search for an ideal: spiritual security in the hereafter, ibid., 794.
    58 E. Uphill, "The Egyptian Sed-Festival Rites", JNES 24, no. 4 (1965).
    59 On the early 4th dynasty curtain box belonging to Queen Hetepheres, wife of Snefru and mother of Khufu.

[^8]:    71 Seated statue in grey porphyroid granite with a large shen encircling its neck, clasped hands on lap. Height 62.5 cm . Louvre N40.
    72 Rijksmuseum van Oudheden, Leiden AST 18, D93.
    73 A relief from the tomb of lynefer, CG 57121, son of Snefru, brother of Khufu, from Dahshur and now in Cairo Museum JE 38564, similarly shows him seated in profile with a staff held out in front. Possibly the Ames staff? He also has a shen-ring

[^9]:    shown worn as a collar.

[^10]:    76 F. Monnier, "The Satellite Pyramid of Meidum and the Problem of the Pyramids Attributed to Snefru", JAEA 3 (2018).
    77 Monnier and Lightbody, The Great Pyramid: Haynes Operations Manual, 36-59.
    78 W.M.F. Petrie, Medum (London: David Nutt, 1892).
    79 Verner, 461.

[^11]:    87
    This is now displayed in the garden area at the entrance of the Egyptian Museum in Cairo.
    88 In later heraldry this is known as a close or overt attitude when applied to depictions of hawks and falcons.

[^12]:    101 Verner, 70; B. Mojsov, Osiris. Death and Afterlife of a God (London: Blackwell Publishing, 2005), 26; I.E.S. Edwards, The Pyramids of Egypt (Middlesex: Penguin, 1979), 269; S. Clarke and R. Engelbach, Ancient Egyptian Construction and Architecture (New York: Dover Publications, 1991), 118.
    102 For a fuller discussion of the literature on this subject, see Lightbody, Egyptian Tomb Architecture. The Archaeological Facts of Pharaonic Circular Symbolism, S1852, 49.
    103 Petrie, Wisdom of the Egyptians, 30.
    104 R.J. Gillings, Mathematics in the Time of the Pharaohs (New York: Dover Publications, 1982), 184.
    105 From the textual evidence, the 'sekeds' used for applying the slopes on site were calculated from the desired base and height dimensions for any pyramid, and not the contrary as has been suggested elsewhere. It has also been proposed that the slope of the pyramid reflected circular proportions because of a coincidence that a seked of $5 \frac{1}{2}$ palms gave this slope when a 7-palm cubit was used, but it is in fact more likely that the circular symbolism neatly fits this seked precisely because the 7-palm cubit was originally chosen to facilitate calculations involving circles. This relationship between the cubit and the circle means that applied problems of practical circular geometry often fit well into the 7-palm royal cubit system, as well as into the seked slope system. It is notable that most of the other slopes of passages, gable vaults, and shafts in the Great Pyramid also appear to have followed whole or half-seked-like slopes, which supports the proposal that this type of system was used in Khufu's era.

[^13]:    bottom being 280 palms this leaves 80 for the height".
    110 V. Maragioglio and C.A. Rinaldi, L’architettura Delle Piramidi Menfite 4. La Grande Piramide Di Cheope, Plates, vol. IV (Rapallo: Tipografia Canessa, 1965), pl. 7; Petrie, The Pyramids and Temples of Gizeh - 1st Edition, 83, xiii. The actual dimensions of the room, with the height measured down to the real floor of the chamber - not to the relatively rough surface layer of paving slabs which is approximately 13 cm thick, are in meters (averaged out across the chamber): 10.47 m length $\times 5.974 \mathrm{~m}$ height x 5.235 m width. Given the tolerances of $0.6^{\prime \prime}$ Petrie quotes, this includes exactly the 80 palms in height that he estimated was the design height deliberately included by the architects, so that the circuits of the north and south sidewalls are indeed 440 palms.
    111 A fragment of a heb-sed relief from Khufu's complex has also been recovered.
    112 There is no clear indication that the so-called Queen's Chamber of the Great Pyramid was built with these symbolic proportions, although the shape of the niche in the west wall may indicate an effort to add a geometric feature with a symbolic dimension. This possibly indicates that the King's Chamber replaced the Queen's Chamber during construction and incorporated additional architectonic symbolism at that time.

[^14]:    116 Menkaure's throne name "established are the kas of Re" can plausibly be related to the Giza site. The three pyramids that had been established there by the start of his reign represented the kas of three pharaohs. By Menkaure's era the pharaohs were also associated with the solar god Re. In the context of encircling symbolism, the fact that Re was represented by a circle with a central point must be considered notable. See further discussion in the current publication, 41.
    117 For example, the pair of panels on the columns from the pyramid of Unas shown below (Figure 5-2). In fact, the lines running over the top of the falcons on this throne are elongated sky glyphs, showing the two halves united.

[^15]:    123 K. Lutley and J. Bunbury, "The Nile on the Move", Egyptian Archaeology 32 (2008), 5.

[^16]:    125 J.P. Allen, Egyptian Art in the Age of the Pyramids (New York: Metropolitan Museum of Art, 1999), 319, 321. NY Met N.A. 1992.2.

    126 Borchardt, 44, 45.

[^17]:    127 A. De Trafford, "The Palace Façade Motif and the Pyramid Texts as Cosmic Boundaries in Unis's Pyramid Chambers", CAJ 17, no. 3 (2007), 279 (recto text 3, 7).
    128 T.A.H. Wilkinson, 210.
    129 P. Posener-Kriéger, "Remarques Sur L’ensemble Funéraire De Neferirkare Kakai À Abu Sir", in Festschrift Für Siegfried Schott, ed. W. Helck (Wiesbaden: Otto Harrassowitz, 1968); idem, The Abu Sir Papyri. Hieratic Papyri in the British Museum, Fifth Series (London: Trustees of the British Museum, 1968).

[^18]:    130 Fragments were recovered from the complexes of Neferefre, Neferirkare-Kakai, and his wife Khentkaus II.
    131 E. Brovarski, "Once More $H r^{\prime}$ Pyramid'?"; R.H. Wilkinson, "The Coronational Circuit of the Walls, the Circuit of the Hnw Barque and the Heb-Sed 'Race' in Egyptian Kingship Ideology", JSSEA 15: 1 (1987); G.A. Gaballa and K.A. Kitchen, "The Festival of Sokar", Orientalia 38 (1969).
    132 Posener-Kriéger, "Remarques Sur L’ensemble Funéraire De Neferirkare Kakai À Abu Sir", 112-120.
    133 Posener-Kriéger, The Abu Sir Papyri. Hieratic Papyri in the British Museum. Shen-rings appear on pls. 31 (fragment B.1), 31A, $27 F, 29 E, 32.7$, and 67 (fragment B). See also the 6 th dynasty representation of an offering table referenced in entry 54 in the catalogue at the end of this work, and the 5th dynasty offering table with shen rings referenced in catalogue entry 50.
    134 Posener-Kriéger, The Abu Sir Papyri. Hieratic Papyri in the British Museum, pls. V\&VA. Louvre E. 25.279 recto, column b. For the example in Fig. 5-4 see pls. VII\&VIIA. Louvre E.25.416b, column i. From the texts, it is clear that both the god's servants and "land-tenant" priests were partaking in this activity. $w^{c} b$ Wab-priests also engaged in the same activities as referenced in other fragments of the Abusir Papyri. See A. Wilson, Pure Ones: The W'b and W'bt from the Old Kingdom to the End of the Middle Kingdom. Masters Thesis (Cairo: AUC, 2014), 33.

[^19]:    139 Lightbody, "The Encircling Motifs of Old Kingdom Avian Themed Pharaonic Vases". This paper was followed in 2018 by further research on the group of vessels, carried out at the Oriental Institute in Chicago and on photographs obtained from the Louvre in Paris. That work revealed additional symbolism integrated into the designs of the vessels studied.
    140 Ziegler, 462.
    141 Sycamore wood was associated with the goddess Hathor, mother/wife of the god Horus. A fragment associated with this vase (181883) carries the glyph for Hathor.

    142 ÄM 18807 Ägyptisches Museum und Papyrussammlung, Berlin.
    143 L. Borchardt, "Ausgrabungen Bei Abusir Januar Bis Juni 1907", MDOG 34 (1907).
    144 D. Arnold, C. Ziegler, and C. Roehrig, Egyptian Art in the Time of the Pharaohs (New York: Metropolitan Museum of Art, 1999), 344-347.

[^20]:    155 Gardiner, Egyptian Grammar: Being an Introduction to the Study of Hieroglyphs, 485 [N 1].

[^21]:    161 R. Shonkwiler, "Sheltering Wings: Birds as Symbols of Protection in Ancient Egypt", 49.
    162 R. Hari, La tombe thébaine du père divin Neferhotep (TT50) (Geneva: Editions de belles-lettres, 1985), 51.

[^22]:    163 J.P. Allen, "Reading a Pyramid," in Hommages à Jean Leclant (Cairo: IFAO, 1994).
    164 De Trafford, "The Palace Façade Motif and the Pyramid Texts as Cosmic Boundaries in Unis's Pyramid Chambers".
    165 PT 221 §198d "for you are Horus surrounded/encircled by the protection of his eye", see R.O. Faulkner, The Ancient Egyptian Pyramid Texts (Kansas: Stilwell, 2007), 49; K. Sethe, Die Altaegyptischen Pyramidentexte Nach Den Papierabdrucken Und Photographien Des Berliner Museums, vol. 1 (Leipzig: J.C. Hinrichs'sche Buchhandlung, 1908), 115. Also c.f. §195e "for he is Horus surrounded/encircled by the protection of his eye".
    166 The glyphs used for the word "encircle" here are also used in the name for a cartouche. The full set of glyphs for the word cartouche have Gardiner's sign list codes: V7-N35-W24-G43-V9.
    167 G. Robins, The Art of Ancient Egypt (Cambridge, Massachusetts: Harvard University Press, 1997), 12.
    168 J.P. Allen, A New Concordance of the Pyramid Texts. Volume IV. Pt 422-538 (Providence: Brown University, 2013), 185. See page headed PT 534, central column includes part 48, 1277c as recorded by K. Sethe.
    169 Faulkner, The Ancient Egyptian Pyramid Texts, 200, 202.
    170 J.P. Allen and P. Der Manuelian, The Ancient Egyptian Pyramid Texts, Writings from the Ancient World (Atlanta: Society of Biblical Literature, 2005), 166, 167.
    171 S.A.B. Mercer, The Pyramid Texts (Toronto: Longmans, Green and Co., 1952), 208-209.

[^23]:    172 J. Osing, "Zu Spruch 534 Der Pyramidentexte", in Études pharaoniques; Hommages à Jean Leclant 1 (Cairo: IFAO, 1994).
    173 lbid., 283.
    174 H. Hays, The Organization of the Pyramid Texts: Typology and Disposition, vol. 1 (Leiden: Brill, 2012), 100.
    175 Ibid., 130, 274, 408.
    176 Erman and Grapow, Wörterbuch Der Ägyptischen Sprache, IV, 491.

[^24]:    177 This interpretation of the first line remains tentative and should be taken with a pinch of salt, however, the connection between large eggs and elite burials is well established in ancient Egyptian archaeology and later funerary texts. C.f. Friedman and Muir.
    178 De Trafford, 275.
    179 Ibid., 278.
    180 A.J. Spencer, "Two Enigmatic Symbols and Their Relations to the Sed Festival", JEA 64 (1978), 55.
    181 T.A.H. Wilkinson, 210.

[^25]:    183 See BM EA 567, line 11; Iskander, J.M. "The Haker Feast and the Transformation", in Studien zur Altägyptischen Kultur 40 (Hamburg: Helmut Buske Verlag, 2011), 137-142; Végh, Z. "Counting the Dead: Some Remarks on the Haker-Festival", in Proceedings of the Tenth Annual Current Researches in Egyptology Symposium, Liverpool 2009 (Oxford: Oxbow, 2010); Anthes. See also the stele of Ikernofret, Berlin Museum 1204 for more details on the Middle Kingdom mysteries of Osiris.
    184 Végh. "Counting the Dead: Some Remarks on the Haker-Festival", 146-147 \& 153-154.
    185 M. Sankiewicz and A. Ćwiek, "The Scene of 'Going Round the Wall' on the North Wall of the Portico of the Birth", in Polish Archaeology in the Mediterranean. Reports 2006 (Warsaw: University of Warsaw, 2008).
    186 R.H. Wilkinson, "The Coronational Circuit of the Walls, the Circuit of the Hinw Barque and the Heb-Sed 'Race' in Egyptian Kingship Ideology", 46; Ritner, 58.
    187 Gaballa and Kitchen.

[^26]:    188 C.f. the $12^{\text {th }}$ dynasty coffin of Khnumhotep 12.182 .131 a , b, and the $13^{\text {th }}$ dynasty coffins of Netnofret \& Ikhet, and Entemaemsaf \& Anon, 32.3.429, 430, 431, 432, all now in the Metropolitan Museum of Art in New York.
    189 W.C. Hayes, Royal Sarcophagi of the XVIII Dynasty (Princeton: Princeton University Press, 1935).
    190 MFA Boston 04.278.1. See also Hatshepsut's two other sarcophagi, now in Cairo, JE 37678 from KV20, and JE 52459, and see C. H. Roehrig, Hatshepsut: From Queen to Pharaoh (New York: Metropolitan Museum of Art, 2005), 188; P. Der Manuelian and C.E. Loeben, "From Daughter to Father. The Recarved Egyptian Sarcophagus of Queen Hatshepsut and King Tuthmose I." Journal of the Museum of Fine Arts Boston (1993); P. Der Manuelian and C.E. Loeben, "New Light on the Recarved Sarcophagus of Hatshepsut and Thuthmose I in the Museum of Fine Arts, Boston", JEA 79 (1993)..
    191 Lid Fitzwilliam E.1.1823 and box Louvre D1.

[^27]:    192 A. Dodson and S. Ikram, The Mummy in Ancient Egypt. Equipping the Dead for Eternity (London: Thames and Hudson Ltd, 1998), 214; Shonkwiler.
    193 MFA Boston 72.4820c.
    194 K.A. Bard, An Introduction to the Archaeology of Ancient Egypt (Oxford: Blackwell Publishing, 2008), 248; C. Roehrig, "The Building Activities of Thutmose III in the Valley of the Kings", in Thutmose III: A New Biography, eds. E. Cline and D.B. O'Connor (Ann Arbor: University of Michigan Press, 2006).
    195 J. Zandee, "Review of S. Schott, Die Schrift Der Verborgenen Kammer in Königsgräbern Der 18. Dynastie", BiOr 18, no. 1-2 (1961), 36-37; E. Hornung, The Ancient Egyptian Books of the Afterlife (Trans. David Lorton) (Ithaca New York: Cornell University Press, 1999).

[^28]:    198 The door was removed from TT100 and is now in the Louvre, C24.
    199 Cairo Museum JE 72273. See S. Hassan, The Great Sphinx and Its Secret: Excavations at Giza 8 (1935-36) (Cairo: Government Press, 1953), 62-63; C. Zivie-Choche, Sphinx: History of a Monument (New York: Cornell University Press, 2002), 60-63, Fig. 12; A.G. Shedid, "Die Stele Des Mentu-Her", in Egyptian Museum Collections around the World 2, eds. M. Eldamaty and M. Trad (Cairo: Supreme Council of Antiquities, 2002).

[^29]:    202 The most notable primary source is the Rhind Mathematical Papyrus collected by the Scottish antiquarian Alexander Henry Rhind in Thebes, now in the British Museum (BM 10057 \& 10058). It dates to c.a. 1,650 B.C. and was probably copied from an original from c.a. 1,850 B.C. See A.B. Chace, The Rhind Mathematical Papyrus (Ohio: Oberlin, 1929); Gillings, 142; T.E. Peet, The Rhind Mathematical Papyrus (Liverpool: The University Press, 1923).
    203 See Lightbody (2008) and the current work.
    204 In the late 19th century, the issue of circular symbolism in the Great Pyramid was addressed by the Astronomer Royal for Scotland, Charles Piazzi Smyth, whose theories on the matter were unfortunately distorted by his religious and nationalist beliefs. It was not until Petrie's survey became available in 1883 that most of Smyth's ideas were effectively debunked, but Smyth's publications continued to confuse the matter and discredited the issue of circular symbolism to some extent in the public mind. Petrie did provide a clear explanation of the architectural phenomenon as he saw it, and the issue was rehabilitated to some extent by the mid-20th century. I.E.S. Edwards was one of those Egyptologists who broadly accepted Petrie's conclusions on the matter. Others did not, and continued to speculate further: c.f. G. Robins and C.C.D. Shute, "Irrational Numbers and Pyramids", DE 18 (1990); idem, "Mathematical Bases of Ancient Egyptian Architecture and Graphic Art", Historia Mathematica 12 (1985); Herz-Fischler, The Shape of the Great Pyramid (Ontario: Wiflred Laurier University Press, 2000).
    205 At the end of the 20th century, the issue of Egyptian scientific capabilities, including the issue of circular symbolism in the architecture of the pyramids, became one of the core issues discussed during the "Black Athena Debate". That discussion addressed the hypothesis published by Martin Bernal, grandson of Egyptologist Alan Gardiner, which advocated a more advanced level of science than had been acknowledged by mainstream Western scholars. See M Bernal, Black Athena: The Afroasiatic Roots of Classical Civilization (London: Free Association Books, 1987). The issue of circular symbolism was drawn into that debate, but the discussions were inconclusive and remained confused because they all referenced the same limited number of philological works rather than the architectural evidence gathered by Petrie and others from the monuments themselves. It is notable, however, that even the most skeptical of the scholars involved was prepared to consider the possibility that the ancient Egyptians had derived a working understanding of the ratio pi. See the discussion in Lightbody (2008), 60.
    206 One of the most widely read and well-developed discussions of the issue of circular symbolism in the Great Pyramid's architecture in recent years was the publication by C. Rossi, Architecture and Mathematics in Ancient Egypt (Cambridge: University Press, 2003). I addressed the counter-arguments made there in my 2008 publication.
    207 See also the Moscow Mathematical Papyrus, the Egyptian Mathematical Leather Roll, and a few other fragmented documents and texts.

[^30]:    209 The relevant mathematical papyrus examples are P. Rhind 41, 42, 43, 48, 50.
    210 Lightbody (2008), 54.
    211 Gillings, 139-153. The circle was equated to the area of a square of $9 \times 9$ with truncated corners. By counting the number of small squares within the truncated square, the area of the equivalent circle could be estimated.
    212 See also C. Rossi, Architecture and Mathematics in Ancient Egypt (Cambridge: University Press, 2003). Also my critique of related works in Lightbody (2008), 56.
    213 Verner, 70; Mojsov, 26; Edwards, 269.

[^31]:    219 Values of $1643.3 \mathrm{~m}, 1642 \mathrm{~m}, 1645 \mathrm{~m}$, and 1646 m are given in the following publications, respectively: J.P. Lauer (1960), 2; idem, (1931), 60 ; Verner (2003), 461; Kemp, 103. Using the average of these values, the proposed relationship is accurate to better than 0.2\%.
    220 L. Cooper, "Did Egyptian Scribes Have an Algorithmic Means for Determining the Circumference of a Circle?", Historia Mathematica 38 (2011): 470.
    221 Lightbody (2016).

